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STRUCTURAL DESIGN STUDY FOR A RADIO TELESCOPE WITH A PARABOLOID REFLECTOR SIX HUNDRED FEET IN DIAMETER

By

Jacob Feld

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Jacob Feld
Consulting Engineer, New York, N. Y.

Accorded the D. B. Steinman Award for Research in Structural Engineering in 1955 by The New York Academy of Sciences

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Introduction

The recent extension of astronomical research by the measurement of radio waves emitted by celestial bodies has created a new type of design problem for structural engineers. The engineering requirements for the construction of radio telescopes are unusual. Such instruments must possess not only stability for static and wind loadings, but also require extreme accuracy of geometric shape and very precise stationary positioning. In addition, such an instrument must be provided with smooth, controllable tracking movements so that it can maintain a continuous focus on any given celestial point. This report is a summary of a study made to determine whether a paraboloid dish 600 ft. in diameter could be designed structurally to provide, in effect, a focusing mirror for wave lengths as short as 10 cm. in all positions covering the entire solar hemisphere, whether such an instrument could be controlled in sidereal motion, and at what costs. Many general ideas have been suggested in the 20-year history of this new subject, and several small dishes, some as large as 84 ft. in diameter, have been built and are in use. However, the much greater size of the proposed 600-ft. instrument necessitated a completely new approach to the problem; the solution proposed here indicates that such an instrument is structurally feasible at a reasonable cost and can be expected to perform within the required tolerances of shape and position under normal wind loadings. This work was performed under contract with the Associated Universities, Inc., New York, N. Y., under the guidance of its Committee on Radio Astronomy.

Conclusions of the Design Study

Generally, a parabolic reflector with a diameter of 600 ft. and a focal ratio of 0.5 (depth of shell, 75 ft.) is structurally feasible, and in any one position can be adjusted to provide and maintain the desired surface tolerances. In any other position, surface deviations must be expected, the degree of such distortion varying with wind and other natural factors. However, part of such variation can be offset automatically by adjustable cables. The remaining deviation from a true parabolic shape will approximate the desired tolerances closely, even with 30-mile winds. Greater deviations must be expected in localized areas of the reflector for short periods of time, but they can be damped out by series of radial ties.

The focal point of the instrument cannot be held at a fixed position in space for all positions of the reflector, and the focal feed tower must be provided with remote-control equipment to permit axial and angular adjustments. Such controls are neither prohibitively expensive nor unduly complicated. The reflector can be attached to a structural frame that permits controlled movement at slow speed and of any desired accuracy about both the horizontal and the vertical axes over complete rotations of 360°. The mounting is sufficiently rigid to ensure accurate maintenance of a given focus under normal winds and operational conditions.

All of the various materials and equipment required are now available, and they are of magnitudes and natures employed previously in the construction and operation of various types of moving structures such as ore-handling bridges and lift and rotating railroad and vehicular bridges. The design and erection of a reflector 600 ft. in diameter is without precedent, but the methods contemplated for doing so are only small extensions of previous practice.

The estimate of the cost of the installation is based upon the actual costs of erected structural steel in tall space structures and of mechanical and electrical equipment of comparable sizes. Allowance has been made for the cost of the foundation on the basis of experience in the general area in which the facility will be constructed. The estimated costs of the major items have been confirmed by independent estimates furnished by contracting firms with experience in heavy mechanical-control equipment and in welded-steel space structures.

FIGURES 1 to 10 illustrate, in some detail, the structural design and assembly of the equipment chosen in consequence of this study of feasibility.

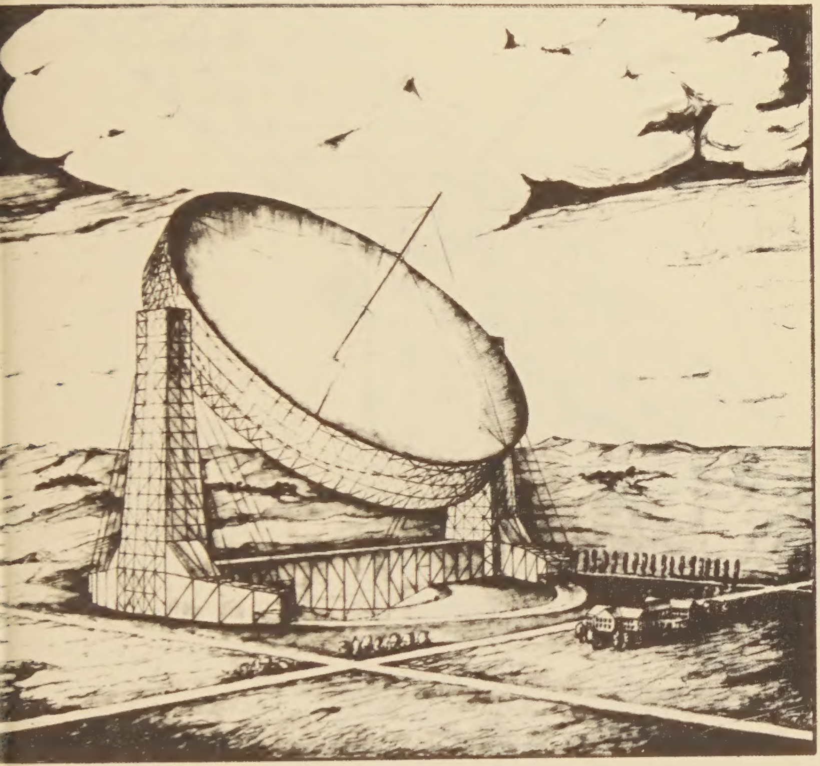

Figure 1. General appearance of the completed installation.

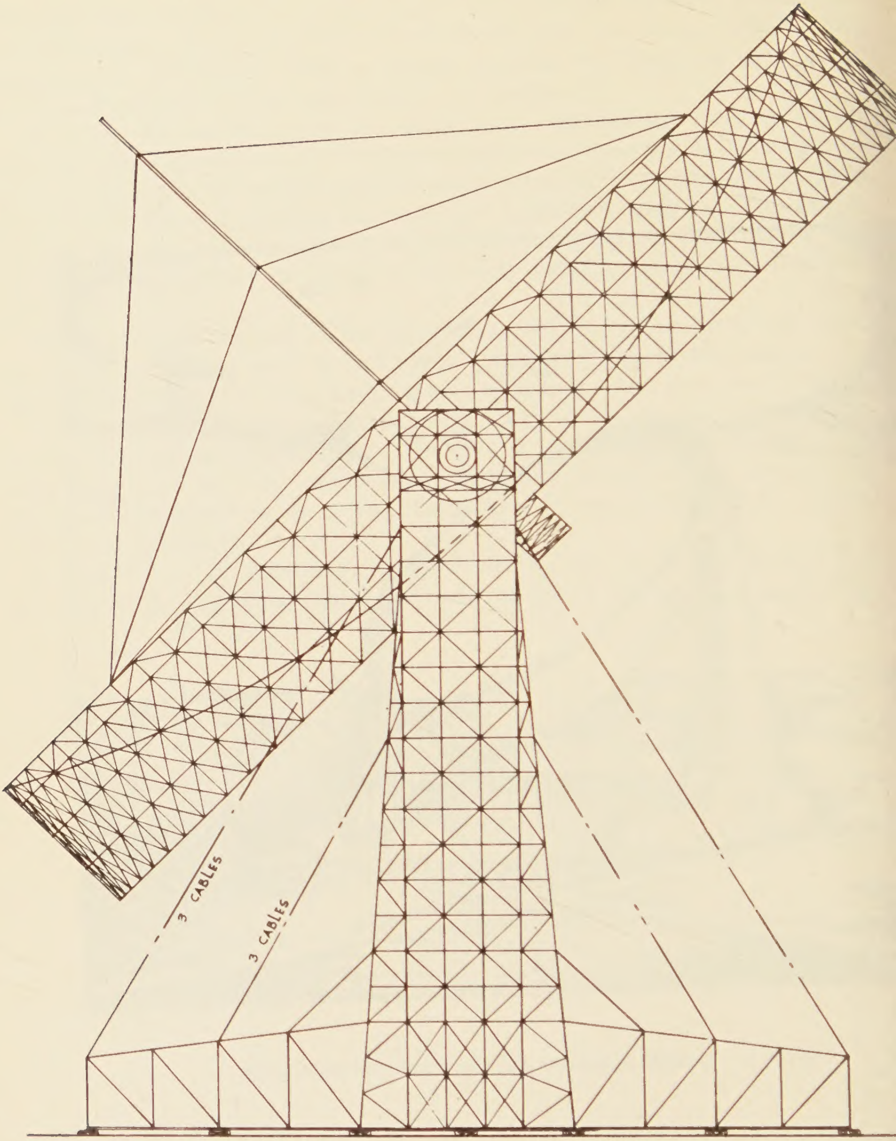

FIGURE 2. Side elevation.

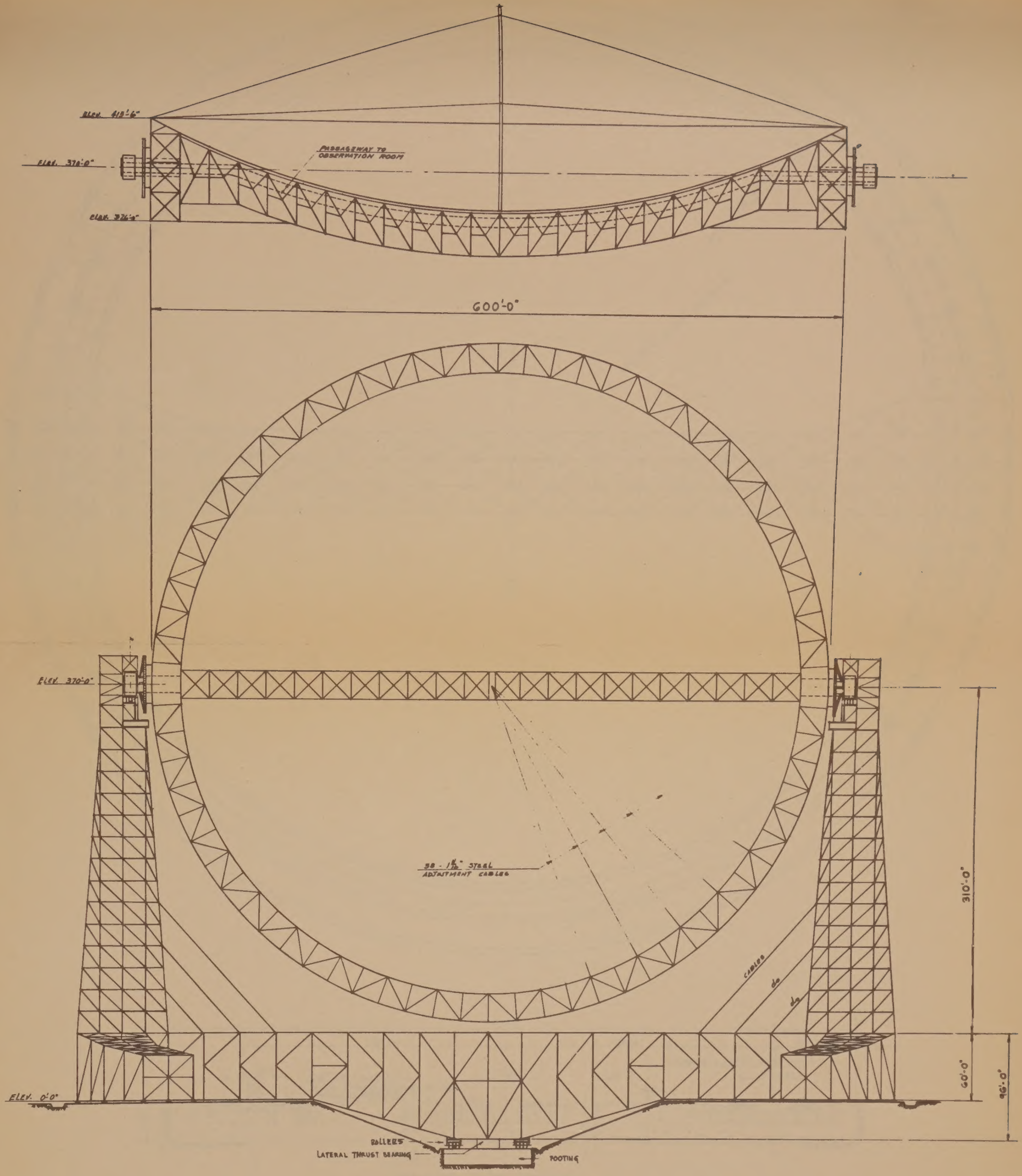

FIGURE 3. Front elevation.

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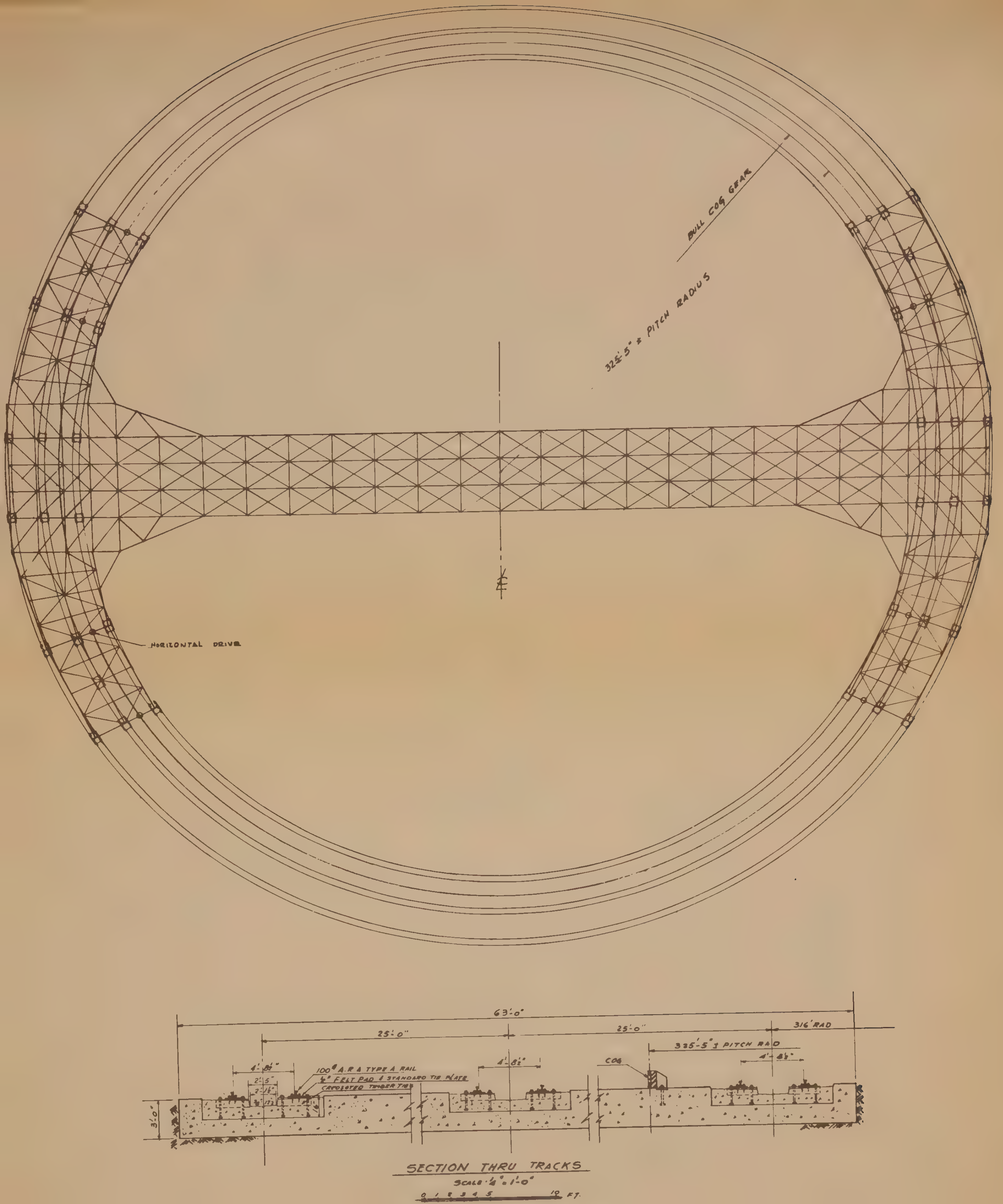


FIGURE 4. Plan of towers and track.

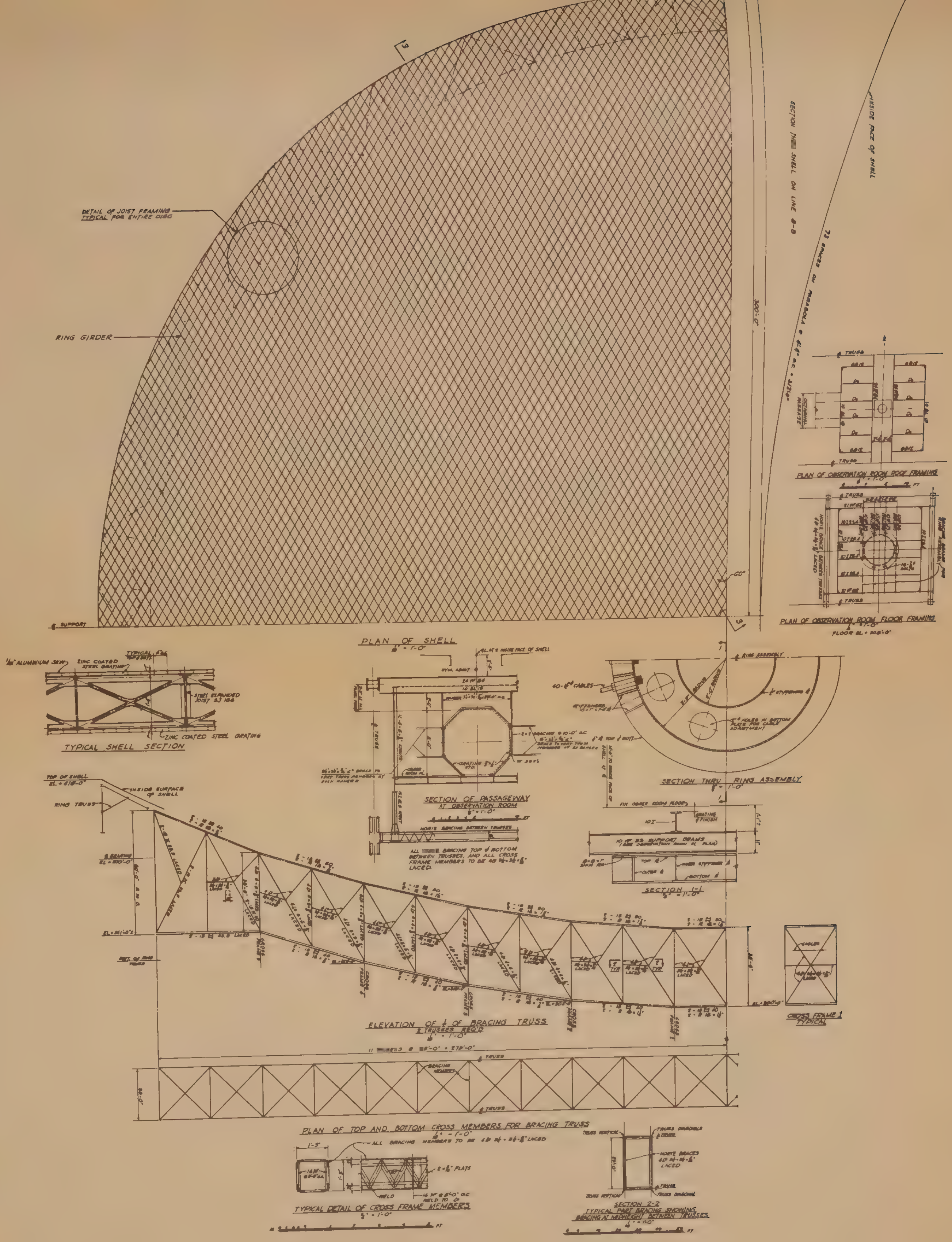

INSIDE FACE OF SHELL
SECTION THRU SHELL ON LINE B-B
73 SPACES ON PARABOLA @ 4'-8" o.c. = 312'-0"
300'-0"
DETAIL OF JOIST FRAMING TYPICAL FOR ENTIRE DISC
RING GIRDER
SUPPORT
60°
PLAN OF OBSERVATION ROOM ROOF FRAMING
PLAN OF OBSERVATION ROOM FLOOR FRAMING
PLAN OF SHELL
1/8" = 1'-0"
TYPICAL SHELL SECTION
1/16" ALUMINIUM SKIN
ZINC COATED STEEL GRATING
STEEL EXPANDED JOIST
SECTION OF PASSAGEWAY AT OBSERVATION ROOM
ALL BRACING TOP & BOTTOM BETWEEN TRUSSES, AND ALL CROSS FRAME MEMBERS TO BE 4∠ LACED.
SECTION THRU RING ASSEMBLY
RING ASSEMBLY
40-CABLES
SECTION 1-1
TOP OF SHELL
INSIDE SURFACE OF SHELL
RING TRUSS
ELEVATION OF 1/2 OF BRACING TRUSS
2 TRUSSES REQ'D
1/8" = 1'-0"
CROSS FRAME 1
CROSS FRAME 2
CROSS FRAME 3
CROSS FRAME 4
CROSS FRAME 5
CROSS FRAME 1 TYPICAL
CABLES
LACED
TRUSS
BRACING MEMBERS
PLAN OF TOP AND BOTTOM CROSS MEMBERS FOR BRACING TRUSS
ALL BRACING MEMBERS TO BE 4∠ LACED
TYPICAL DETAIL OF CROSS FRAME MEMBERS
SECTION 2-2
TYPICAL PART BRACING SHOWING BRACING AT MIDHEIGHT BETWEEN TRUSSES
TRUSS VERTICAL
TRUSS DIAGONAL
HORIZ. BRACES
FT

FIGURE 5. Details of shell and bracing truss.

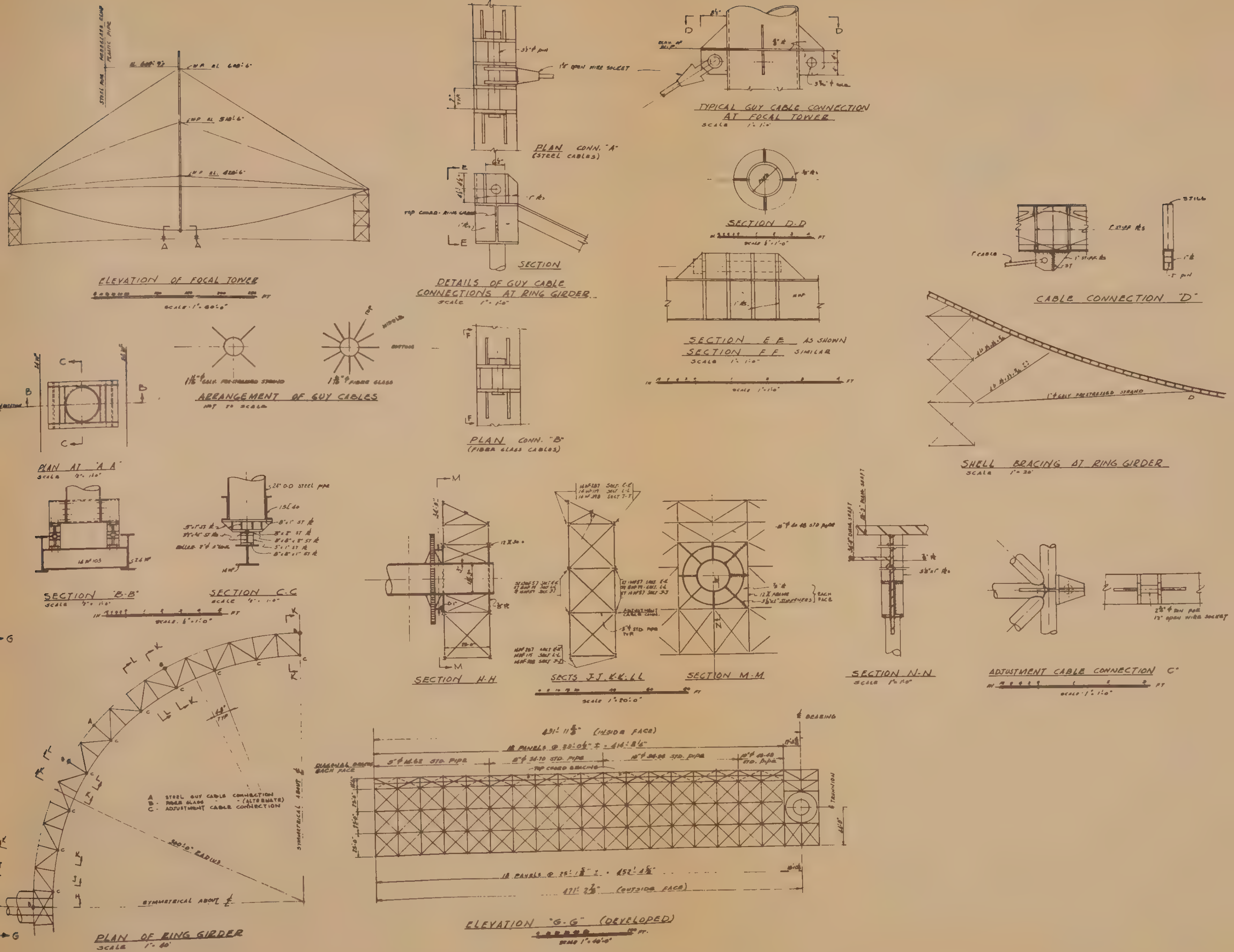
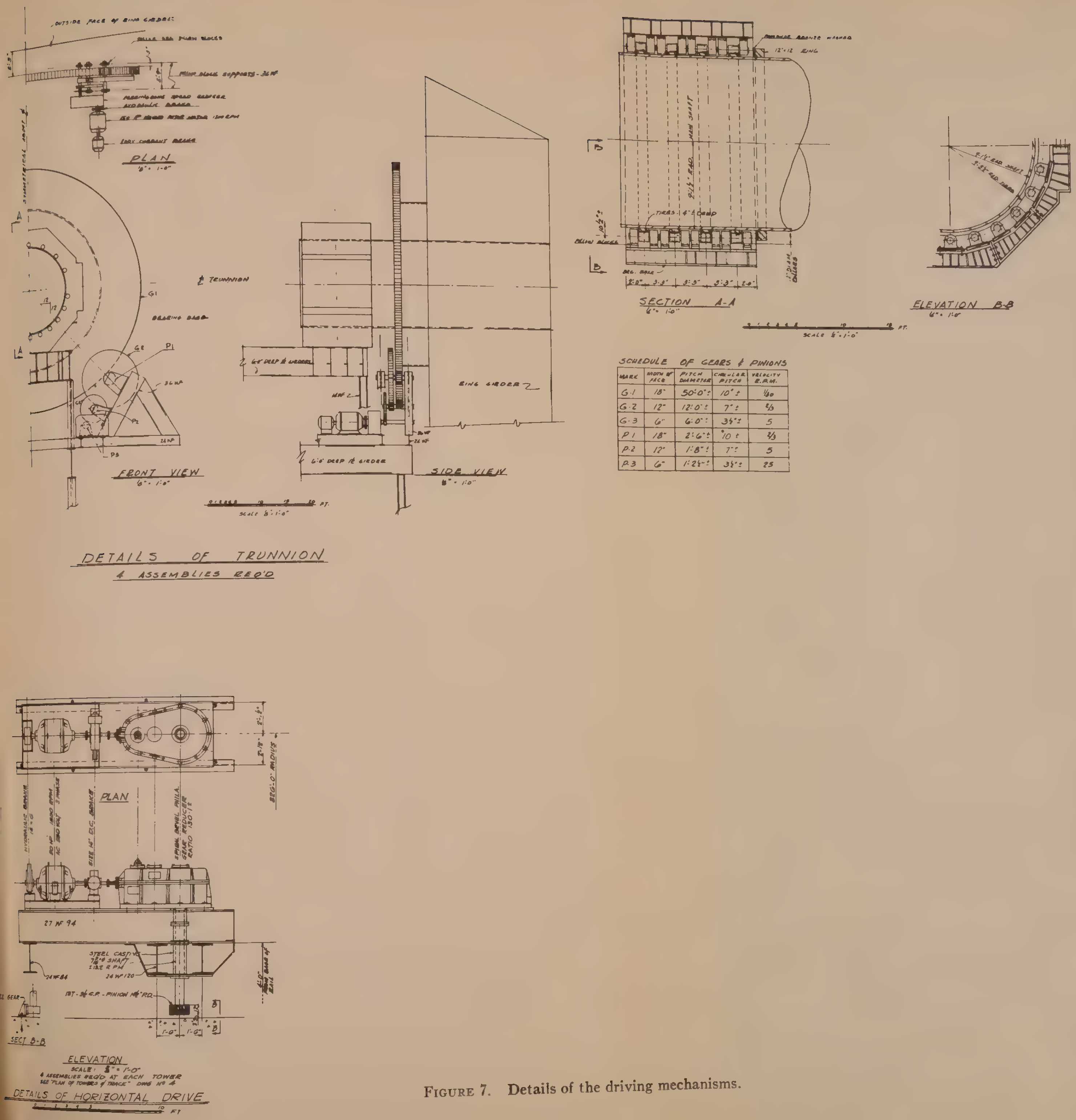


FIGURE 6. Details of ring girder and focal tower.



SCHEDULE OF GEARS & PINIONS

MARK	WIDTH OF FACE	PITCH DIAMETER	CIRCULAR PITCH	VELOCITY R.P.M.
G-1	18"	50'-0"±	10"±	1/60
G-2	12"	12'-0"±	7"±	2/3
G-3	6"	6'-0"±	3½"±	5
P-1	18"	2'-6"±	10"±	2/3
P-2	12"	1'-8"±	7"±	5
P-3	6"	1'-2½"±	3½"±	25

FIGURE 7. Details of the driving mechanisms.

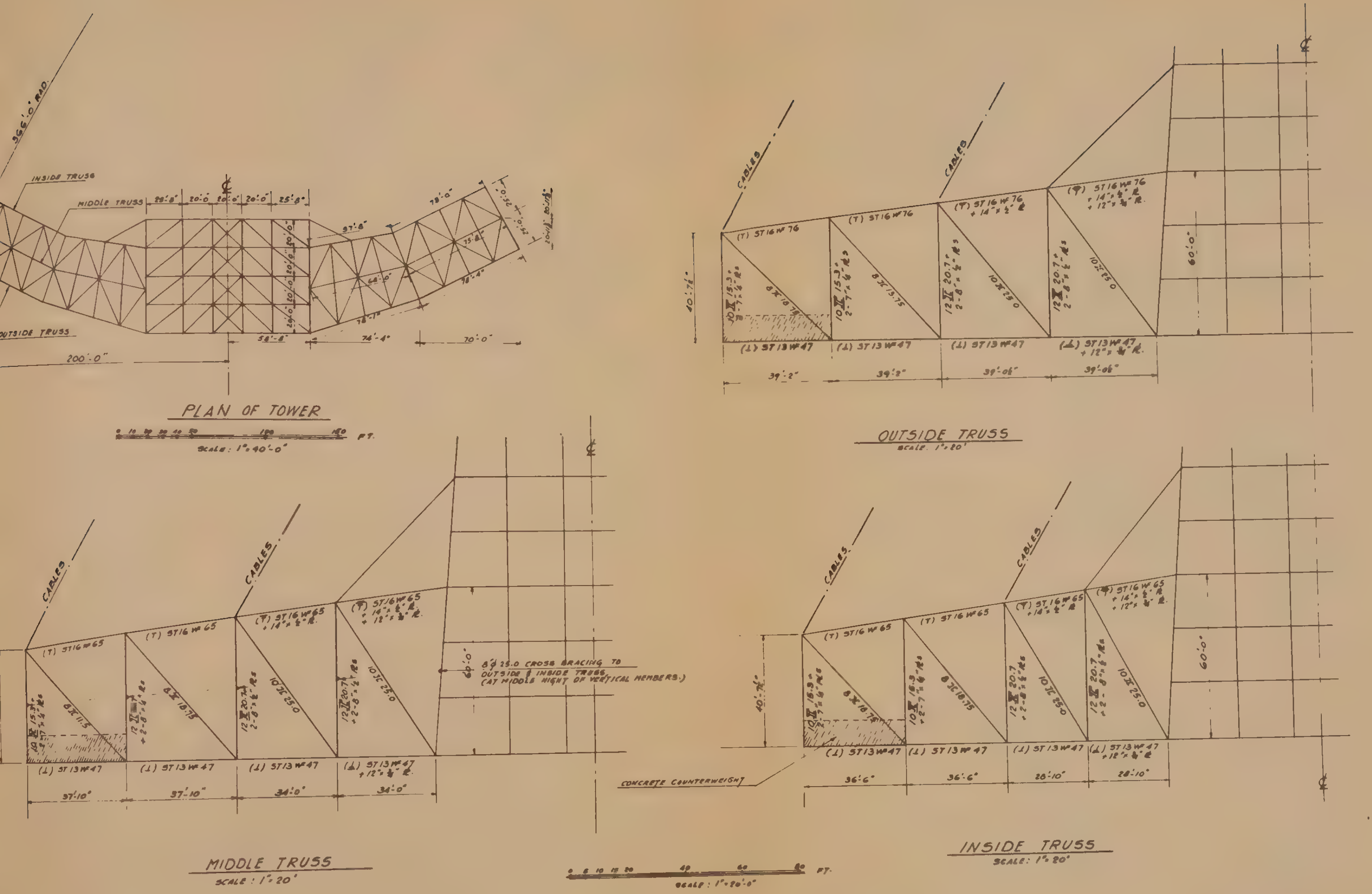


FIGURE 8. Wing trusses for the towers.

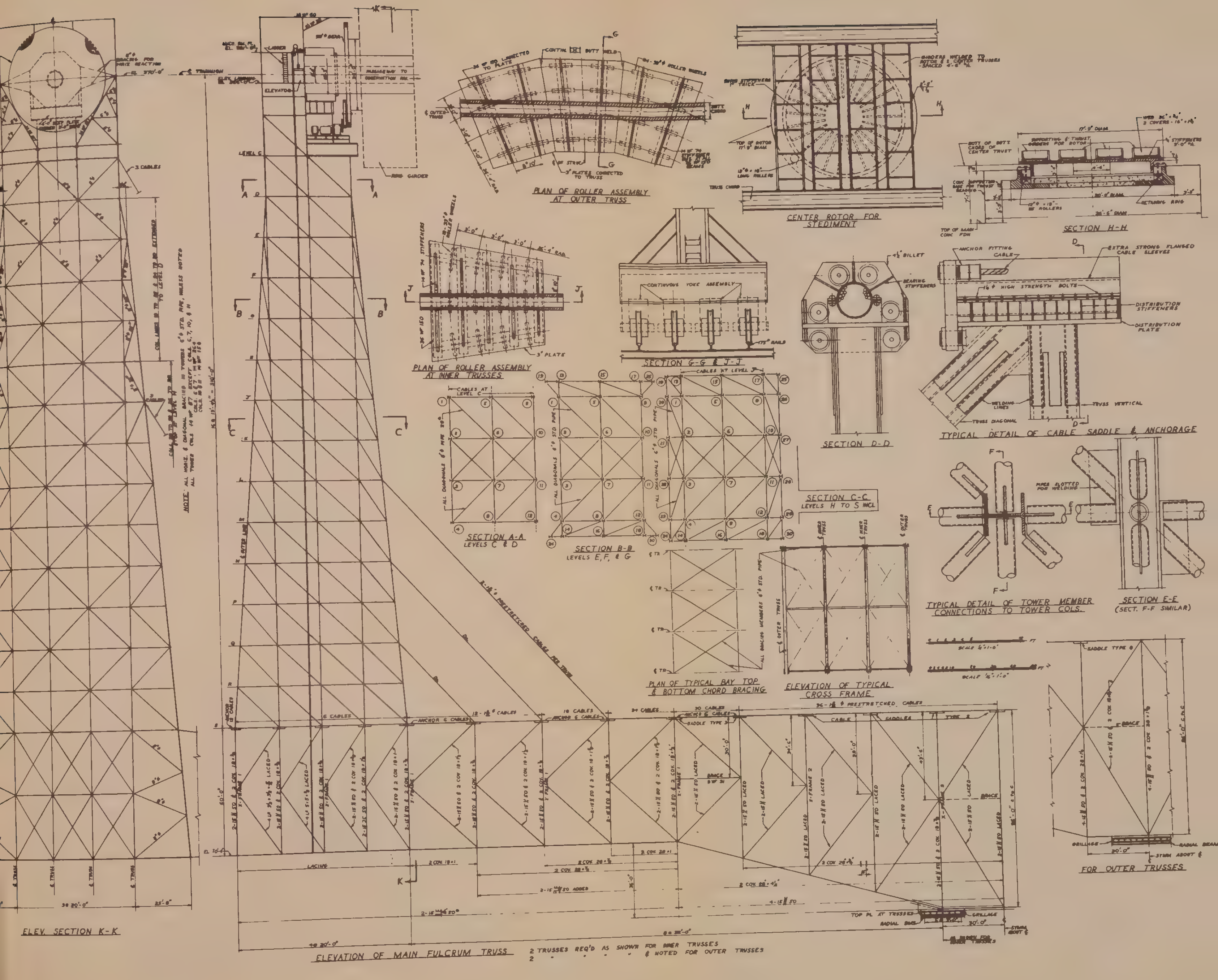


FIGURE 9. Details of towers, cross truss, and support.

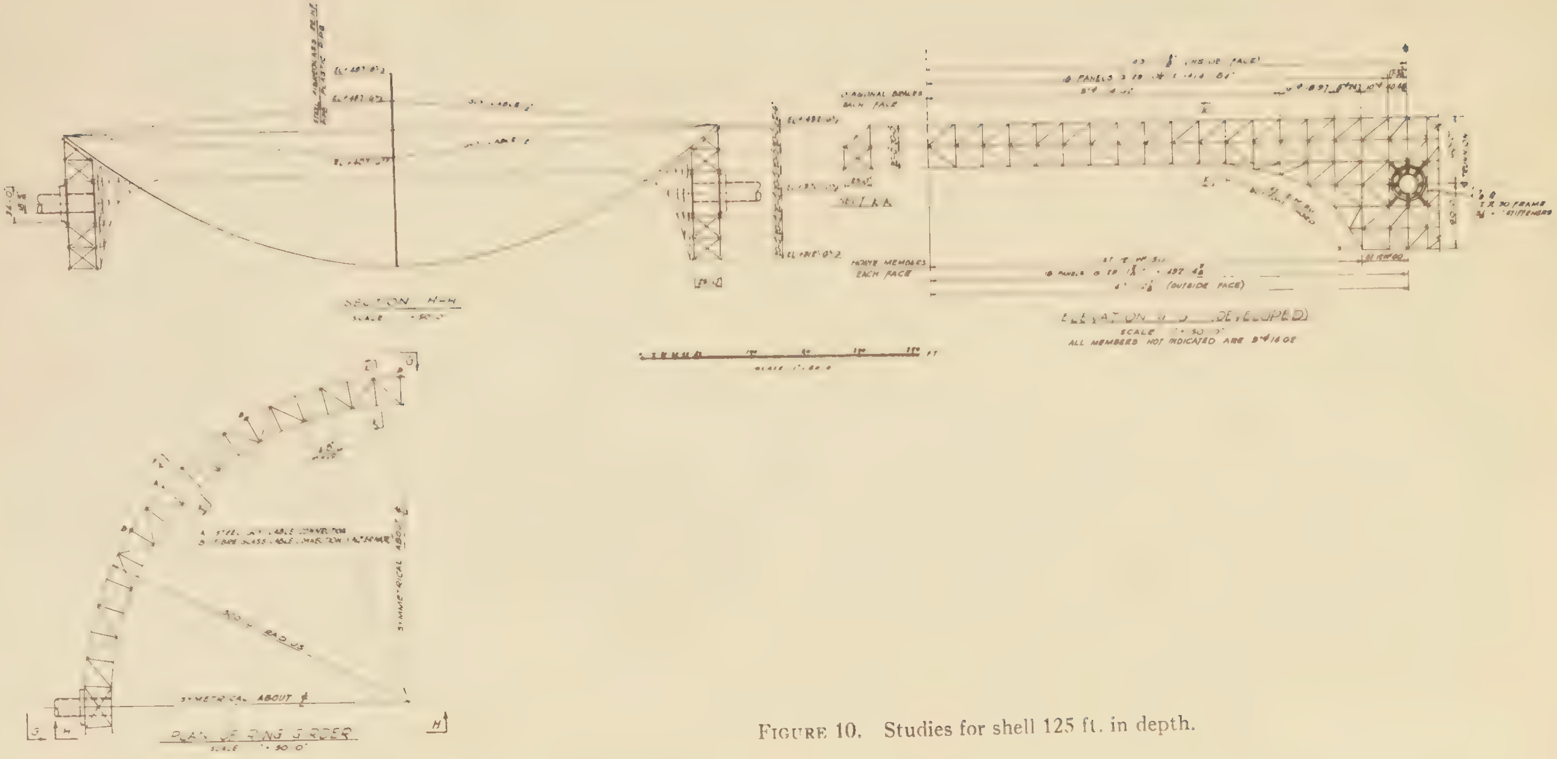


FIGURE 10. Studies for shell 125 ft. in depth.

General Requirements and Chosen Solutions

The basic problem is presented as an outline of the desired requirements for the construction of such an instrument, setting forth the tolerances of shape and the specifications for performance. The important items are listed below, and the solutions chosen for the problems presented by the basic design are described as follows:

Focal ratio. The basic element of the instrument is a parabolic reflector with a ratio of focal length to diameter of aperture of from 0.35 to 0.5, the larger ratio being preferred if the antenna-feed equipment mounted at the focal point can be held in position to the desired tolerances.

Surface tolerances. Deviation from a true parabolic shape by not over ¼ in. is desired, but for a 600-ft. aperture a deviation of 1 in. for the full aperture and ⅝ in. for half of the aperture would be acceptable.

Surface materials. Either a continuous or an open-mesh metal surface is permissible, with permanent electrical continuity and gaps or openings of not more than ¼ in. The choice of the material for the surface will be determined, to a great extent, by the necessity for protection against the concentration of visible and infrared energy at the focus.

Feed at the focal point. Focal-point equipment weighing 500 lb. is to be kept at a desired tolerance of ±⅛ in. for any position of the reflector even when there are winds of as much as 30 mph. The equipment is to be supported by a nonmetallic tripod or quadripod frame. If the tolerance desired is not feasible, remote-control positioning equipment must be provided.

Sky coverage. The instrument should be able to focus upon any point in the entire celestial hemisphere. In addition, it should have the ability to point below the horizon to facilitate calibration of the receiver; preferably, it should be possible to turn the reflector upside down. This requirement necessitates an altazimuth mounting that, it is recognized, is less expensive than an equatorial mount. If mechanical difficulties prevent an altazimuth mount, restricted sky coverage of ±120° about the polar axis and vertical motion from 5° below the pole to 5° below the southern horizon would be acceptable.

Pointing and tracking accuracy. It should be possible to focus the instrument on any point in the sky for a 600-ft. diameter aperture with a maximum tolerance of 7 sec. (⅛ in.) of arc and a preferred tolerance of 2 sec. ($\frac{7}{200}$ in.).

Sluing rate. The desired rates are 30° min. for fast movement and 5°/min. for slow movement. With gear-controlled operation along a circle of 350-ft. radius, these rates correspond to movements of 183 ft./min. (2.08 mi./hr.) and 30.5 ft./min. (0.35 mi./hr.).

The time allotted for this feasibility study was only two and one half months, because a general decision, as well as an estimate of approximate cost, was required by July 11, 1955, with a final report to be submitted by July 20, 1955. With no precedent available, tentative decisions as to basic design procedures and forms of structure and operation were necessary, logically developing into a complete design that would become the basis of a preliminary cost estimate. Where the development of the plan indicated that changes in the framework would be needed to control deformations of the reflector, such changes were made. Further study of the design undoubtedly will show areas of possible and permissible reduction in weight and cost.

Once a decision was made that the requirements for sky coverage could be met only by an altazimuth mounting, the feasibility study was divided into the following parts: (1) the disc; (2) the focal-point support mast; (3) the support and movement of the disc about a horizontal axis; (4) the support towers for lateral and vertical loadings; (5) the cross trusses to concentrate the entire weight on a center bearing area; (6) the movements of the entire frame about a vertical axis; (7) the center bearing; (8) foundations and trackage; and (9) miscellaneous facilities.

(1) *The disc.* Preliminary studies of paraboloid discs of 600-, 500-, and 400-ft. diameters with focal-length-to-diameter ratios of 0.5, 0.35, and 0.30 indicated only a 50 per cent variation in shell membrane and ring- or edge-perimeter stresses, and also in the limiting, crippling, or buckling stress. The best conditions are to be found in the smallest and deepest sizes; the worst are in the largest and shallowest sizes. The shallower discs require deeper ring girders to help support the shell in the horizontal condition; conversely, the shallower discs have greater strength in the vertical position. The surprisingly small variation made it seem likely that a diameter of 600 ft. with a focal length of 300 ft., the shell being only 75 ft. deep, would introduce no problems more serious than those that would be encountered in the other cases. Consequently, the detail study is based upon the 600-ft. diameter and a focal-length ratio of 0.5.

To maintain minimum distortions, the shell should have uniform rigidity and, to reduce costs of the supports and the operating mechanisms, the dead weight of this component must be kept to a minimum. The studies indicate that no economy would result from the use of lightweight metal supporting members because of their low modulus of elasticity and their higher unit cost as compared to those of steel. The choices made are as follows:

(a) A thin sheet of aluminum, perforated with $\frac{1}{8}$-in. holes on 4-ft. centers, treated with a tan-colored stain on both faces. This surface shows very little reflection of infrared and visible wave lengths (about 4 per cent), and the undersurface will radiate the absorbed energy.

(b) A steel grating with a 4-in. × 4-in. pattern, coated with zinc either electrically or by metalizing, to which the aluminum skin can be stud-welded and weld-connected to the frame below.

(c) A welded frame of expanded steel joists, set in a 60° diamond pattern with the long axis vertical when the disc is vertical, the top and bottom chords fully butt-welded, the member lengths arranged in lamella array 3 ft. 9 in. on center (o.c.), and all steel to be coated with zinc or aluminum to eliminate the need for painting. The open-web form will help materially in reducing wind-gust and eddy effects.

(d) A steel grating with a 4-in. × 4-in. square pattern, similar to the top grating layer and welded to the frame above.

(e) A circular ring girder of sufficient strength to balance stresses of the shell membrane and rigid enough to absorb some of the deflection of the shell when it is in the horizontal position. In the vertical position, the shell will be more rigid than the ring girder and will support part of its weight.

(f) A steel-framed chamber under the center of the shell area to form an observation room connected to each main tower by an octagonal mesh-encased walkway. It also acts as the base of the mast.

(g) A system of high-strength prestressed steel-wire cables radially connecting the ring girder with the framework of the center chamber, at which point the tension of each cable can be adjusted to provide for correction of the shape of the shell as influenced by the ring girder, and also as reserve resistance in the general plane of the shell against any deformation that might be caused by local vibration. These cables are not part of the stress-design requirements.

(h) A cross-truss frame under the shell, but connected to the framing of the center chamber and to the ring girder at the location of the main supporting shafts. This truss frame will be less rigid than the shell, but it is designed to transfer a horizontal thrust of approximately 6 per cent of the total weight of the shell, such as might be caused by seismic action, into the bearings at either tower, without calling upon the shell or the ring girder to transfer such lateral force, whether it resulted from wind gusts, operational maladjustments, or extraneous shock.

(2) *Focal-point support mast.* With the focal point protruding 225 ft. above the rim of the disc, any tripod or quadripod framework support becomes a major structure with either considerable flexibility or such bulk that large areas of the disc would be shadowed by it. Consequently, study was directed to the feasibility of using a guyed mast, and this potentiality developed into the following ideas:

(a) A cylindrical mast made of sections of hollow preformed tubes of about 6 ft. in exterior diameter, held together by prestressed cables run through axial holes, the assembly held, by a system of guys, against wind and other lateral forces as well as gravitational force when the mast is away from a vertical position. It is entirely feasible to make the shell of plastic reinforced with glass fiber and the prestressed cables and the guys of protected glass fiber. If this were done, there would be no metal in the entire assembly. However, the cost of these materials is several times that of steel, and equal strength with smaller deformation can be obtained from the use of steel pipe 2 ft. in diameter.

(b) As a more economical solution I suggest a mast constructed of 24-in. standard steel pipe, the top 30 ft. consisting of a plastic section that would not interfere with reception by the dipole unit mounted at the focal point. The small area at the center of the disc covered by this mast is substantially the same area as that shielded by any other method of support for the instruments. Glass-fiber guys can be used with the steel mast, but the flexibility of the mast is about five times that to be expected if high-strength steel cables were used as guys. The latter are recommended as most advisable if their use is practicable within the limitations imposed by the operation of the instrument.

(c) Computations of deformation indicate that the position of the focal point cannot be held in space within the ⅛ in. tolerance even with perfect fabrication and with precision adjustment in any given orientation of the mast. The top section of the mast must have freedom of motion, within limits not exceeding several inches, in the direction of the mast axis and also by a rotation about an axis parallel to the supporting axis of the shell. The device that controls this motion can be actuated either by two small electric motors or by hydraulic pistons. The objections to each of these systems must be weighed; the electric wire connection along the mast as against a small hydraulic tube, almost 300 ft. long, that would be maintained under pressure. Control of the adjustment could be done from the observation room; considerable study and charting of positions of the focal point under the various possible orientations of the mast and shell will be required.

(3) *Support and movement of the disc about the horizontal axis.* The disc is supported structurally by large hollow shafts rigidly connected to the ring girder and seated in the roller-bearing raceways completely surrounding the shafts. A 50-ft. diameter bull gear is connected to act as the transmission for rotational movements to the same shafts, and as close to the ring girder as clearances will permit. With the shaft located on the axis of gravity of the entire shell assembly, the power required under static conditions is very small. To control movements under unequal wind loads acting on the exposed portion of the shell, chiefly when the disc is in a vertical position, the motors and operational facilities were designed for an unbalanced load of 10 psf on 25 per cent of the area acting 150 ft. above the axis. To move the disc at the maximum speed of a 180° rotation in 15 minutes of time, with such unbalanced force acting continuously (an almost impossible set of conditions), would require two 150-H.P. motors at each end. The connection of the motors to the gear of the bull wheel is effected through a reducer-gear box and a series of gears and pinions. With smaller unbalanced loads on the shell, correspondingly faster speeds of rotation are possible. None of the equipment needed is unprecedented or, indeed, unusual.

The desired speed and positioning control cannot be obtained from single-traction operation, since the initial starting friction is so much larger than the friction of normal operation, and instantaneous stopping likewise is impossible. The first solution was to provide duplicate equipment in opposite-hand design, always running one set against the resistance of the other. The same advantages would result from the use of two separate power and gear units, placed symmetrically about the bull gear and equipped with DC generators and Eddy current brakes to level out the rates of motion.

(4) *The support towers.* Structurally, the support towers are similar to bridge towers of not-unusual size, and their vertical and lateral loads are of the order of magnitude encountered in medium-sized lift bridges. The framework is of conventional pattern. To provide further latent resistance to sudden gusts of wind or similar forces, cable guys must be added in directions tangential to the circular movement and fastened to the tangential base of the truss of each tower. Dead-weight loads are insufficient to

eliminate possible uplift tendencies under some extreme wind conditions. To prevent this, concrete counterweights are added at the ends of the truss bases.

(5) *Cross-truss main support.* To avoid possible changes in the main horizontal axis that might result from the settling of the two main towers because of changes in load beneath them, it was considered necessary to support both towers on a cross frame that is to be pinned to a center footing. Supporting all the loads on this frame is an expensive item. The truss consists of two cantilevers, each with a span of over 350 ft., with a load of about 5000 tons at each end. Some means of holding the two towers in specific relative position is essential, and the cross frame serves this purpose. High shear stresses in the frame will result in appreciable deflections under the loading. Most of the load results from the initial steel erection, and the frame can be built to balance this computed deflection. To permit adjustment after erection and, at the same time, to take advantage of the high permissible stresses of prestressed steel cables, which are almost five times those of normal steel, the top chord of the frame is made up of such cables. In effect, the frame will be a heavy double-cantilever bridge truss, equal in depth and section to the largest bridges of this type. If solid rock foundations are available at the chosen site, the reliance on uniform base support under the towers in all positions will permit a considerable saving in the weight of the frame by eliminating the requirement that all the load be carried by it. It was considered unnecessary to impose the resultants of wind loadings on the cross frame, since the amounts of such pressure normally existing are relatively small and will be borne completely by the base trusses under the tower. This arrangement is efficient and economical because of the great leverage of the resistance developed by it.

(6) *Movements about a vertical axis.* With all static loads balanced on a cross frame that is mounted on a center pivot, rotation of the instrument about this pivot is effected easily by means of powered railroad car trucks running on standard railroad tracks. Under ideal conditions, with no wind or other lateral load, this device will have little traction unless counterweighted. Even with weight added, the starting and stopping resistances will not provide the uniform and closely controlled movements necessary to comply with desired tracking and positioning tolerances. The proper solution appears to be the use of a continuous cog gear with traction positive at all times and with close synchronization of position of the opposite towers by electrical interconnection. Since temperature and possible soil variation may affect the relative position of the cog perimeter and the operations of the motor mounted on the tower base, it was decided to place the cogs on a vertical face and to have the operating gear press against the circular gear. Standard railroad car equipment is to be issued, 4- and 6-wheel trucks carrying motor units with high- and low-speed propelling gears. The ring gear is to be set on a circle of 652 ft. in diameter, the drive mechanism located under the center of the towers, and torsional forces kept to a minimum. Four 50-HP motors are required under each tower to provide movement at the rate of 52.7 ft./min., or 39 min. of time for a rotation of 360°. Reduction

to slow speed from the normal speed, which is only 6/10 miles hour, is effected easily. It does not seem advisable to move this large mass any faster.

(7) *Center bearing.* The full weight of the disc, the towers, and the cross frame is supported on the center bearing, and the pressure upon it resulting from vertical load transfer must be equal for any position of the disc. Horizontal forces also must be transmitted to the foundation. This transfer is accomplished by a system of large alloy-steel roller wheels of high strength and hardness that will run on crane rails connected to the foundation. The axles are connected by yokes to a series of grillage beams rigidly connecting and supporting the 4 trusses of the cross frame. The rails are bent in 4 concentric circles. In all positions, a total of 96 rollers is directly below the 4 trusses. The foundation load under the corresponding supporting areas, distributed to the underlying soil through a concrete mat 90 ft. in diameter, is 4 tons/sq. ft. The quality of the soil should be such that no plastic deformation will result from the continuous cycle of loading and unloading to such intensity, and that elastic deformation will not exceed a small fraction of an inch. These requirements for support of the foundation are not too difficult to provide.

(8) *Foundations and trackage.* The foundation of the center support must be a concrete base on solid soil or rock. The size of the footing required is not unusual; by sinking it below the normal ground level, better soil conditions can be expected. The need for such excavation results from the shape of the main cross frame which, for reasons of economy, is deeper at the center support than at the towers. Railroad trackage is of the type used in underground rapid-transit railroads, consisting of heavy standard rail on subties of creosoted oak set into a concrete roadbed. The subgrade for the roadbed must be of satisfactory rigidity, but it need not be solid rock. Standard materials are used for the entire trackage except for the ring gear for the drive control. This gear can be constructed in identical sections, flame-cut from solid steel stock and connected in a manner similar to that used in rail splices.

(9) *Miscellaneous facilities.* Access to all critical points of the telescope will be provided. Each main tower will have a push-button controlled passenger elevator with a speed of 200 ft. min., as well as a stairway for emergency use. A passage is provided through the pipe shafting and by way of an enclosed octagonal passageway from each high operations deck into the observation room directly below the focal-point mast. Since the disc can be rotated 360°, it is not necessary to provide a ladder within the mast, although this is possible. The observation room is enclosed completely, and it can be equipped with seats on two axis pivots for automatic adjustment as the position of the telescope changes. A connecting passageway within the ring girder around the entire perimeter of the disc is possible, but it may be too dangerous to be practicable. A passageway is provided between the two tower bases to afford a connection between the bottom of the two elevators and the room enclosing the center pivot. Also, from this passageway at the center of the cross frame there is a hydraulic-lift platform for access to the focal point when the disc is inverted. The framework is of

uch large scale that there is room for almost any additional facility; even visitors' galleries and living quarters for personnel can be provided. The additional weights resulting from such changes will have very little effect on he present design of the structure and its operational equipment.

Calculation Procedure for the Structure

These calculations evaluate stresses and deformations in the shell, as well as movements of the focal point within reasonable range of accuracy, under assumed loadings and considerations of physical properties of materials used. As a first approach, stresses in the direction of the meridian and the ring have been calculated for a dome shell supported along its circumference. The formulae for symmetrical loadings are shown in page 189. Where a shell is discontinuous, as at the ring, bending that deforms the shell into damped waves along the meridians is introduced. The shell constant k and the wave length λ depend upon the radius of curvature and the thickness of shell; the wave length increases with the thickness and the radius of a shell. The ordinates of all damped waves, representing factors such as induced bending, ring and meridian forces, and distortions caused by discontinuities, have similar ratios for all shells (page 189). Shells of different rise-to-length ratios for length $L = 600$, 500, and 400 ft. have been calculated for a circumferentially supported dome, and their stresses are shown on pages 190 to 194. The make-up of the shell is detailed on page 196, and the resulting unit stresses are summarized on page 197. A preliminary design indicated the choice of the shell members as consisting of expanded joists (SJ 166), spaced 4 ft. 4 in. on center (o.c.), a diagonal joist (SJ 102), and 2 layers of steel grating, the upper one supporting the skin. A shell of uniform thickness with the same moment of inertia has a depth $\delta = 4.22$ in. (page 196). The tabulation of properties and shell stresses for different types of shells show that higher height-to-length (H/L) ratios are preferable because of lower membrane stresses. For identical shell make-up, the damping of disturbances also occurs in shorter waves.

For instance, let us compare a shell in which $L/H = 4.8$ with $L/H = 8$ and $L = 600$ ft.*:

$$
\begin{aligned}
&\frac{S_K(4.8)}{S_K(8)} = 1.083 \text{ (skin area)} \\
&\frac{T_{1K}(4.8)}{T_{1K}(8)} = 0.76 \qquad \frac{T_{2K}(4.8)}{T_{2K}(8)} = 0.64 \\
&\frac{\lambda(4.8)}{\lambda(8)} = 0.83 \\
&\frac{\sigma_{CR}(4.8)}{\sigma_{CR}(8)} = 1.05
\end{aligned}
\tag{1}
$$

---

* See page 189 for definitions of terms.

With a total load of [dead load $(DL) = 10 +$ live load $(LL) = 10]$ $w_T = 20$ psf, the factor of safety for buckling in the direction of the meridian,

$$n = \frac{\sigma_{CR}}{\frac{T_1 + T_2}{2A}}$$ for the shell $L = 600$ ft. and $H = 75$ ft. $L/H = 8$ at the ring would be

$$\frac{T}{A} = \sigma = \frac{335 \times 20}{1.76} = 3800 \qquad n = \frac{6800}{3800} = 1.79 \,. \tag{2}$$

considering that actually at the connection of the shell to the ring the stress at the meridian will be zero or negative (tension), the critical region for buckling will occur about 70 ft. away from the edge of the shell.

$$\begin{array}{llll} \text{Here:} & y = 230 & & R_1 = (1 + 0.146)^{3/2} \times 600 = 740 \\ & x = \ \ 44 & & \\ \tan\phi = \sqrt{\frac{x}{f}} = \sqrt{\frac{44}{300}} = 0.382 & & & \sigma = \frac{335 \times 20}{1.76} = 3800 \\ & \sigma_{CR} = 7800 & & n = \frac{7800}{3800} = 2.06 \end{array} \tag{3}$$

For the shell, $L = 600$ ft., $H = 125$ ft., $L/H = 4.8$, the critical region for buckling will occur about 50 ft. away from the ring.

$$\begin{array}{ll} y = 250 & R_1 = (1 + 0.495)^{3/2} \times 360 = 650 \\ x = \ \ 90 & \\ \tan\phi = \sqrt{90/180} = 0.707 & \sigma = \frac{T_1}{A} = \frac{235 \times 20}{1.76} = 2640 \\ \sigma_{CR} = 8800 & n = \frac{8800}{2640} = 3.35 \end{array} \tag{4}$$

Antisymmetrical loads occur when the shell is inclined toward the horizontal and superimposed on symmetrical loads that gradually become smaller until the shell is in the vertical position, where the symmetrical loads are reduced to zero.

Using Dischinger's[1] work as a basis, the stress calculation has been made for a spherical shell as a substitute for the paraboloid, with the shell supported at the two towers. The stress vectors are no longer in the direction of the meridian and of the ring, but in their projection parallel and perpendicular *to the axis* of rotation (pages 198 to 203).

Stresses are shown for two shells, each having a length-to-height ratio $L/H = 8$ and $L/H = 4.8$.

For instance, for $L/H = 8$, $\alpha = 90°$, $\beta = 0°$, $N\beta = 528w$, $N\alpha = 126w$ that is, an average of $N\beta = N\alpha = 654/2 = 327w$

where $w = 20$ $N\beta = N\alpha = 6540$ psf as compared with $T_1 = T_2 = 335w$, figured for the shell as a supported dome.

As seen from the graphs (FIGURE 11), the shell with a smaller ratio L/H is preferable because the stresses are lower and more evenly distributed in he direction of the ring and the meridian.

Furthermore, when it is in a horizontal position, the shell with 75-ft. rise, at the given weight of 10.5 psf as shown on SHEET 8, can support only its own dead load (DL), while any additional load must be supported by the ring, including live load (LL) and the weight of the ring itself. In the case of a shell with a rise of 125 ft., the dead load of 10.5 psf can be carried by the shell, and the ring must carry only its own load (pages 224 and 225).

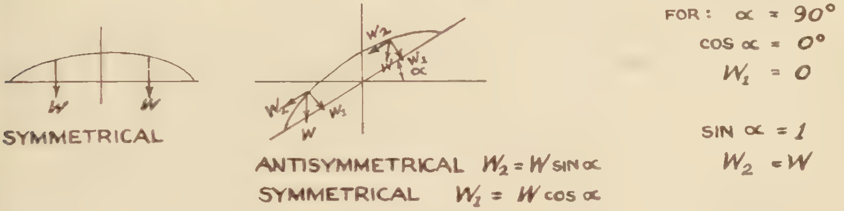


FIGURE 11

Calculations of stress and deflection based on the approximation of shell sections to beams with straight-line stress distribution are shown on pages 208 to 218.

At lines of discontinuity of the shell, as at the ring, where the flow of membrane forces is interrupted, shear forces perpendicular to the surface are introduced, with consequent bending of the shell. This flexing causes additional superimposed meridian and ring forces, causing primary distortions in the direction of the meridian and secondary distortions in the direction of the ring. All of these forces and distortions extend over the face of the shell in the form of damped waves, the amplitudes and wave lengths of which depend upon the radii and thickness of the shell (page 189).

The formulae for bending forces as shown on pages 204 and 205 allow the calculation of disturbances of the shell resulting from differences of temperature between the shell and the ring or differences in strain between the shell and the ring caused by superimposed loads.

Bending stresses caused by a 20° F. difference in temperature between the shell and the ring are shown for the 75-ft. shell on page 206. The bending moment for the fully restrained shell is 4.7 times the maximum moment for the hinged shell, but the maximum distortion for the restrained shell that occurs about 9 ft. $= \frac{\lambda}{8}$ away from the ring is only 60 per cent of the distortion of the hinged shell, or about 15×10^{-4} radians.

The total expected movements of the focal point for a bowl with a 75-ft. rise and a length of 600 ft. are shown on pages 219 to 221. Part of these movements is counterbalanced during erection. The movements have several sources:

(1) *The deflection of the paraboloid, with a 75-ft. rise, acting as a curved beam supported on two points.* Based on straight-line distribution of stresses,

a maximum deflection of the crown of the shell, when in horizontal position, would amount to a differential in height (δH) of 3 in.; as regards the shell with a 75-ft. rise (see page 224) $\cong$ 50 per cent of the load must be carried by the ring, which means an increase by 20 per cent ± of the total load of 3507^K, as shown on page 208. The deflection δH therefore increases to 3.6 in., compared with the ring deflection for the 50 per cent load, which was calculated as 4.3 in. An average of $\delta H = 4$ in. can be expected. To this must be added the deflection of the crown caused by membrane stresses and changes in temperature; for a 20-lb. total LL and $\delta t = 40°$ F., $\delta H = 0.4$ in. The change of focal length (f) is then $\delta f = 4\delta H = 4 \times 4.4 = 18$ in. Shells with higher rises carry a proportionally greater part of their load without assistance from the ring: the 125-ft. shell, in particular, will carry almost 100 per cent of its own weight. The changes of focal length are proportionally smaller and, for the 125-ft. shell, will amount to $4 \times {}^{125}/_{75} \times 0.4 \cong 3$ in. No bending deflection will result from temperature and membrane stresses because the ring carries no part of the shell.

DL and LL cause lateral movements of the focal point in the magnitude of 4 to 5 in., if the shell is in horizontal position and braced with glass-fiber guys. For the shell in vertical position, a lateral movement of the shaft caused by wind may amount to 2 in. (page 219). With cable guys of prestressed steel, these movements are reduced to only ${}^{5}/_{24}$ in., or 20 per cent of these values.

(2) *Movements of the focal point caused by distortions.* These distortions occur mainly at the intersection of ring and shell, and they extend in damped waves with wave length λ. The maximum distortion is for the hinged condition at the edge of the shell and for the restrained condition $\frac{\lambda}{8}$ ft. away from the shell.

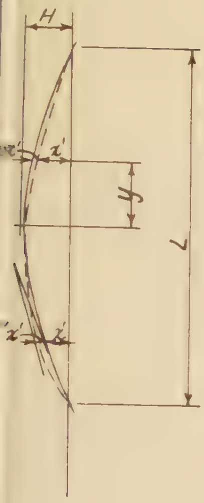
In the horizontal position, DL and LL plus 20° F. difference in temperature between the shell and the ring cause a scattering of the focal points in a circle of about 1.8 ft. in diameter for the hinged condition and 1.08 ft. in the restrained condition. Generally, no distortions will occur outside this rim of $\frac{\lambda}{2}$ ft. width. In the vertical position, the distortions from DL plus LL at the ring will be smaller, and temperature differences alone will be of significant magnitude. For instance, for 20° of difference in temperature, the dislocation of the focal point in a circle 1.17 ft. in diameter for hinged condition and 0.7 ft. for the restrained condition can be expected.

However, distortions occur over the whole surface in the vertical position. Their magnitude, as shown in FIGURE 12, is without appreciable influence.

In positions of the shell between the horizontal and the vertical, the components of forces and deformations add vectorally, as shown on pages 215 to 218.

Conclusion

It can be said that a paraboloid shell with a 600-ft. diameter and a rise of 75 ft. is feasible and will always give good results over about 80 per cent of



$$\partial \tan\phi = \partial\phi = \Delta = \frac{2}{L\sqrt{1-\frac{x'}{H}}}\,dx'.$$

FOR INSTANCE: $\frac{x'}{H} = \frac{3}{4}$ WHERE $H = 75'$

$x' = 56.25'$

$y = 150.0$

$$\eta = \frac{2y}{L} = 0.5$$

$$dx' = \frac{G_{\text{MAX.}x}\;X'}{E} = \frac{6000\times 56.25}{3\times 10^{7}} = 112\times 10^{-4}\ \text{FT.}$$

$$\Delta = \frac{2}{600\sqrt{\frac{1}{4}}}\,dx' = \frac{4}{600}\,dx' = 0.75\times 10^{-4}$$

$$df = \frac{y\,\Delta}{(\sin 2\phi)^{2}} \qquad y = 150$$

$$\tan\phi = \frac{4H\eta}{L} = 0.5\times 0.5 = 0.25$$

$$\phi = 14^{\circ}$$

$$\sin 2\phi = \sin 28^{\circ} = 0.47$$

$$(\sin 2\phi)^{2} = 0.22$$

$$df = \frac{150\times 0.75\times 10^{-4}}{0.22} = 510\times 10^{-4} = 0.61\ \text{IN.}$$

FIGURE 12

ts surface. This percentage can be increased by using shells of smaller L/H atio than 8, preferably one as low as 4.8.*

The study of stresses and deflections of the mast supporting the focal point s shown separately for steel and glass fiber guys on pages 226 to 230. Uni- orm temperature changes will not affect the accuracy of the instrument page 231). The effects of the cable guys on the ring girder are evaluated n pages 232 to 238, and the design of the ring girder under various loadings nd positions is detailed on pages 239 to 245.

The main shafts are designed for various combinations of wind loading nd torsional forces. The worst conditions are summarized on pages 246 to 49. The major members and stresses of the supporting towers and frames re given on pages 250 to 264.

Calculated deformations. For the shell, 600 ft. in diameter and 75 ft. deep nd having its focal point supported 225 ft. above the outer perimeter of the hell, the following computed deflections and deformations have been valuated:

Shell in horizontal position. The shell, acting independently of the ring, ill deflect 1.4 in. at its center, both from the action of the beam, from tresses on the membrane, and from its dead weight. This can be compensated completely by cambering and adjusting the shell an additional 0.13 in. for each 1 psf of live load over the entire area. For example, 1 in. of ice over the entire area will produce a deflection of 0.65 in. of the shell and a displacement of the focal point of 4 × 0.65 or 2.60 in. from the original position and 1.95 in. from the new position.

---

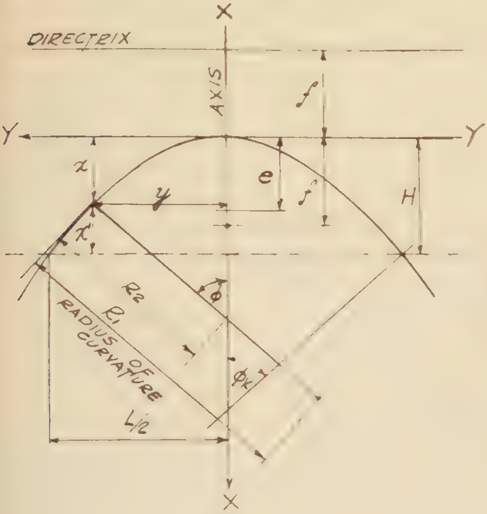
* The deformation and focal-point movements described above are calculated on the ssumption that the shell and ring girder operate as separate and distinct structural elements. Joint-action analysis indicates a considerable reduction in these values.

When acting integrally with the ring, the shell will be deflected at the center because of the action of the beam, the stresses of the membrane, and the dead weight of shell and ring. This deflection can be compensated completely by cambering the ring to a maximum of about 6 in. and adjusting the shape of the shell during assembly an additional 0.13 in. for each 1 psf of live load over the entire area, as is to be done for the shell acting alone.

Wind forces have no appreciable effects on the shell when it is in a horizontal position. A wind load of 30 mph on the mast will cause a displacement of not over $\frac{1}{8}$ in.

When the shell is in the vertical position, acting independently of the ring, it will be deflected about $\frac{1}{4}$ in. from dead weight alone if the entire weight of the ring is carried by the shell. The deflection will not exceed $\frac{3}{4}$ in. These deformations can be compensated for by the automatic adjustment of the radial cables. A uniform horizontal wind of 30 mph will cause a deformation of the depth of the shell of about 0.4 in. Such a distortion will result in a change of the focal point of 1.6 in. Cross winds in the direction of the plane of the shell have little effect. A wind load of 30 mph on the mast, plus the effect of dead weight upon it, will cause a deflection of $1\frac{1}{8}$ in., the vertical component being 1 in. at the focal point. This deviation must be compensated for by the use of the adjusting mechanism of the focal point.

GENERAL PARABOLOID GEOMETRY



$$y^2 = 2px = 4fx$$

WHERE f = FOCAL LENGTH

$$y = 2\sqrt{fx}$$

$$\frac{dy}{dx} = \sqrt{\frac{f}{x}} = \frac{1}{\tan\phi}$$

$$\tan\phi = \sqrt{\frac{x}{f}}$$

$$S = 4\pi\sqrt{f}\ \frac{2}{3}\left|\sqrt{(f+x)^3}\right|_0^x$$

WHERE S = SURFACE AREA

$$= 8.37\sqrt{f}\ \left|\sqrt{(f+x)^3}\right|_0^x$$

CENTROID OF THE SURFACE AREA

$$Se = \int 2\pi x y\, ds$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{2f}{y} \qquad \left(\frac{dy}{dx}\right)^2 = \frac{f}{x}$$

$$ds = \sqrt{1+\frac{f}{x}}\ dx$$

$$Se = 2\pi\int_0^x 2\sqrt{fx}\,(x)\sqrt{\ +\frac{f}{x}}\ dx = 4\pi\sqrt{f}\int_0^x x\sqrt{x+f} = \frac{8\pi}{15}\sqrt{f}\ \left|(3x-2f)\sqrt{(x+f)^3}\right|_0^x$$

$$\text{WHEN } x=H,\quad e = 0.2\,\frac{\left|(3x-2f)\sqrt{(x+f)^3}\right|_0^H}{\left|\sqrt{(x+f)^3}\right|_0^H}$$

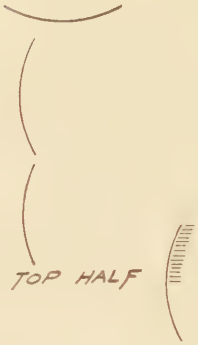
DESIGN LOADINGS

a. DEAD LOAD + 10# w

b. DEAD LOAD + 10# w

c. DEAD LOAD + 30# w

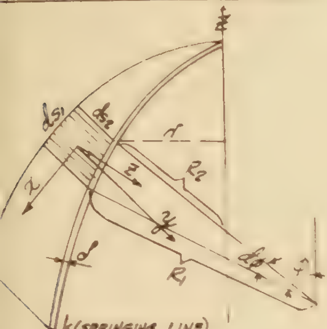
d. DEAD LOAD + 10# w ON TOP HALF



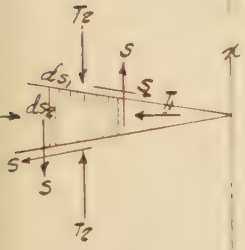
DESIGN STRESSES & PROPERTIES

	STRUCTURAL CARBON STEEL ASTM A 7	HIGH STRENGTH STEEL	ALUMINUM TYPE 14S-T6	CABLES GLASS FIBER	CABLES PRESTRESSED STEEL
YIELD POINT MIN. (PSI)	33 000	50 000	53 000		
MODULUS OF ELASTICITY (PSI)	29/30 000 000	30 000 000	10 600 000	5 000 000	24 000 000
POISSON RATIO	3/10		1/3		
COEFFICIENT OF EXPANSION PER DEGREE F	0.000 0063	0.000 0063	0.000 012		
THERMAL CONDUCTIVITY	0.12		0.53		
TENSION (PSI)	20 000	27 270	22 000	100 000	100 000
COMPRESSION (PSI)	$17000-0.485\frac{\ell^2}{r^2}$ FOR $\frac{\ell}{r}<120$ $\frac{18000}{1+\frac{\ell^2}{18000r^2}}$ FOR $\frac{\ell}{r}>120$	$22500-0.574\frac{\ell^2}{r^2}$ FOR $\frac{\ell}{r}<125$ $\frac{22500}{0.5+\frac{\ell^2}{13440r^2}}$ FOR $\frac{\ell}{r}>125<200$	SECTION B ASCE TRANSACTION PAPER Nº 2536[5]		
SHEAR (PSI)	13000	14500	10 000		
BEARING (PSI)	30000		30 000		

ME SHELL STRESSES



ELEVATION



PLAN

$$T_1 = \frac{S\,w}{2r\pi\sin\phi}$$

WHERE w = LOAD PER FT.^{2}

$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = w\cos\phi$$

$$T_2 = \left(w\cos\phi - \frac{T_1}{R_1}\right)R_2$$

$$\underline{\text{RING FORCE}} = \frac{S\,w}{\tan\phi\;2\pi}$$

$$\sigma_{cr} = \frac{E\delta^2}{R_1\sqrt{3(1-\nu^2)}} = \frac{E\delta}{1.65\,R_1} = \frac{1.44\times10^6\,\delta}{R_1\ \text{ft}}\ \text{psi}$$

(CRITICAL BUCKLING STRESS AT WHICH LOCAL BUCKLING MAY OCCUR. SEE Pg 496, ELASTIC STABILITY BY TIMOSHENKO)[4]

WHERE $\nu = 3/10$ (POISSON RATIO)
δ = SEE SHEET 8 & 8A
$E = 29\times10^6$ psi

$$R_1 = \frac{(1+\tan^2\phi)^{3/2}\,L^2}{8H}$$

$$R_{1k} = \frac{L}{8}\,\frac{(\alpha^2+16)^{3/2}}{\alpha^2} \quad \text{WHERE } \alpha = \frac{L}{H}$$

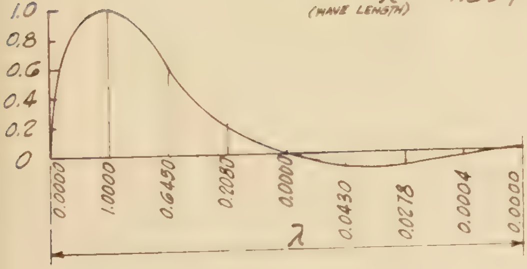
$$R_2 = \frac{r}{\sin\phi}$$

$$K = 0.925\,\frac{R_1}{\sqrt{R_2\,\delta/2}}$$

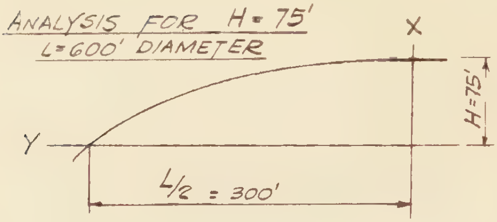
(SHELL CONSTANT)

$$\lambda = 4.89\sqrt{R_2\,\delta}$$

(WAVE LENGTH)



ANALYSIS FOR $H = 75'$
$L = 600'$ DIAMETER



$$y^2 = 2px = 4fx$$

$$f = \frac{y^2}{4x} = \frac{(300)^2}{4 \times 75'} = 300'$$

$$y^2 = 1200x$$

$$y = 34.65\sqrt{x}$$

$$S = 8.37\sqrt{300}\left|\sqrt{(300 + x)^3}\right|_0^x$$ WHERE $x = 75'$

$$S = 302\,000^{\square'}$$

$$\tan\phi = \sqrt{\frac{75}{300}} = \frac{1}{2}$$

$\phi = 26.55^\circ$ SIN $\phi = 0.447$ $\cos\phi = 0.894$

$$\text{RING FORCE} = \frac{302\,000\ w}{\tan 26.55^\circ \times 2\pi} = +96\,000\,w$$

$$T_1 = \frac{302\,000\ w}{2\pi \times 300 \times \sin 26.55^\circ} = 355\,w$$

$$T_2 = \left(w\cos 26.55^\circ - \frac{355w}{R_1}\right)R_2 = \left(0.894\,w - \frac{355w}{838}\right)670 = +312\,w$$

WHERE $R_2 = \dfrac{300}{\sin 26.55^\circ} = 670$

$$R_{1(\phi=k)} = (1 + 0.500^2)^{3/2} \times \frac{600^2}{8 \times 75} = 838$$

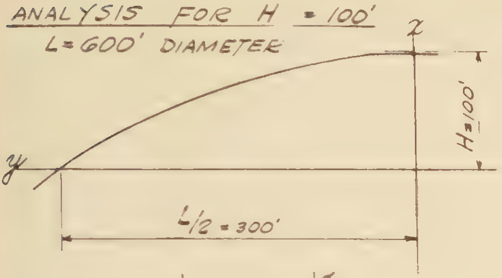
$$k = 0.925 \times \frac{838}{\sqrt{670(0.164)}} = 75$$

$$\lambda = 4.89\sqrt{670\,(0.328)} = 72'$$

$$\sigma_{cr} = \frac{1.44 \times 3.92 \times 10^6}{838} = 6800 \text{ psi}$$

ANALYSIS FOR $H = 100'$

$L = 600'$ DIAMETER



$y^2 = 2px = 4fx$

$f = \frac{y^2}{4x} = \frac{(300)^2}{4 \times 100} = 225'$

$y^2 = 900x$

$y = 30\sqrt{x}$

$S = 8.37\sqrt{225}\left|\sqrt{(225+x)^3}\right|_0^x$ WHERE $x = 100'$

$= 313\,000^{\square'}$

$\tan\phi = \sqrt{\frac{100}{225}} = \frac{2}{3}$

$\phi = 33.75^\circ$ $\sin\phi = 0.555$ $\cos\phi = 0.832$

RING FORCE $= \frac{313\,000\,w}{\tan 33.75^\circ \times 2\pi} = 74\,800\,w$

$T_1 = \frac{313\,000\,w}{2\pi \times 300 \times 0.555} = 300\,w$

$T_2 = \left(w \cos 33.75^\circ - \frac{300\,w}{R_1}\right)R_2 = \left(0.832\,w - \frac{300\,w}{780}\right)540 = 242\,w$

WHERE $R_2 = \frac{300}{0.555} = 540$

$R_{1\,(\phi = K)} = \left[1 + \left(\frac{2}{3}\right)^2\right]^{3/2} \times \frac{600^2}{8 \times 100} = 784$

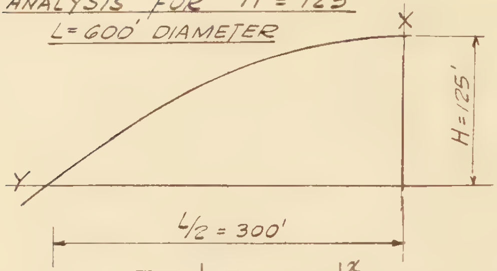
$K = 0.925 \times \frac{780}{\sqrt{540\,(0.164)}} = 78$

$\lambda = 4.89\sqrt{540\,(0.328)} = 64'$

$\sigma_{CR} = \frac{5670 \times 10^3}{780} = 7020$ psi

ANALYSIS FOR $H = 125'$

$L = 600'$ DIAMETER



$$y^2 = 2px = 4fx$$

$$f = \frac{y^2}{4x} = \frac{(300)^2}{4 \times 125} = 180'$$

$$y^2 = 720\,x$$

$$y = 26.8\sqrt{x}$$

$$S = 8.37\sqrt{180}\left|\sqrt{(180+x)^3}\right|_0^x \quad \text{WHERE} \quad x = 125'$$

$$= 327\,000^{\square'}$$

$$\tan\phi = \sqrt{\frac{125}{180}} = 5/6 = 0.833$$

$$\phi = 39.8^\circ \qquad \sin\phi = 0.640 \qquad \cos\phi = 0.767$$

$$\text{RING FORCE} = \frac{327000\ w}{\tan 39.8^\circ \times 2\pi} = 62200\ w$$

$$T_1 = \frac{327\,000\ w}{2\pi \times 300 \times 0.640} = 270\ w$$

$$T_2 = \left(w \cos 39.8^\circ - \frac{270\,w}{R_1}\right)R_2 = \left(0.767w - \frac{270\,w}{795}\right)468 = 200\ w$$

$$\text{WHERE} \quad R_2 = \frac{300}{0.640} = 468$$

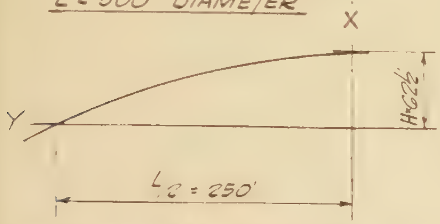
$$R_{1(\phi = k)} = \left[1 + \left(\tfrac{5}{6}\right)^2\right]^{3/2} \times \frac{600^2}{8 \times 125} = 795$$

$$K = 0.925 \times \frac{795}{\sqrt{468\,(0.164)}} = 86$$

$$\lambda = 4.89\sqrt{468\ (0.328)} = 59'$$

$$\sigma_{CR} = \frac{5670 \times 10^3}{795} = 7100\ \text{psi}$$

ANALYSIS FOR $H = 62\frac{1}{2}'$
$L = 500'$ DIAMETER



$$y^2 = 2px = 4fx$$

$$f = \frac{y^2}{4x} = \frac{(250)^2}{4 \times 62.5} = 250'$$

$$y^2 = 1000\,x$$

$$y = 31.6\sqrt{x}$$

$$\pounds = 8.37\sqrt{250}\,\Big|\sqrt{(250+x)^3}\,\Big|_0^x \quad \text{WHERE } x = 62.5 \quad = 209\,000^{\square'}$$

$$\tan\phi = \sqrt{\frac{62.5}{250}} = \frac{1}{2}$$

$$\phi = 26.55^\circ \qquad \sin\phi = 0.447 \qquad \cos\phi = 0.894$$

$$\text{RING FORCE} = \frac{209\,000\,w}{0.500 \times 2\pi} = 65500\,w$$

$$T_1 = \frac{209\,000\,w}{2\pi \times 250 \times 0.447} = 298\,w$$

$$T_2 = \left(w \cos 26.55^\circ - \frac{298\,w}{R_1}\right)R_2 = \left(0.894\,w - \frac{298\,w}{700}\right)560 = 263\,w$$

$$\text{WHERE } R_2 = \frac{250}{0.447} = 560$$

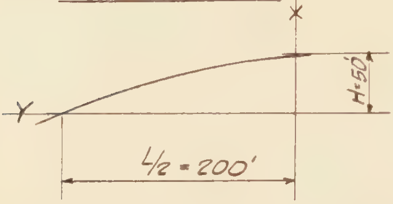
$$R_{1(\phi=k)} = \left[1 + \left(\tfrac{1}{2}\right)^2\right]^{3/2} \times \frac{(500)^2}{8 \times 62.5} = 700$$

$$K = 0.925 \times \frac{700}{\sqrt{560\,(0.164)}} = 69$$

$$\lambda = 4.89\sqrt{560\,(0.328)} = 65'$$

$$\sigma_{cr} = \frac{5670 \times 10^3}{700} = 8100 \text{ psi}$$

ANALYSIS FOR $H = 50'$
$L = 400'$ DIAMETER



$$y^2 = 2px = 4fx$$

$$f = \frac{y^2}{4x} = \frac{(200)^2}{4 \times 50.0} = 200'$$

$$y^2 = 800\,x$$

$$y = 28.3\sqrt{x}$$

$$S = 8.37\sqrt{200}\,\left|\sqrt{(200+x)^3}\right|_0^x \text{ WHERE } x = 50 = 133\,500^{\square'}$$

$$\tan\phi = \sqrt{\frac{50}{200}} = \tfrac{1}{2}$$

$$\phi = 26.55^\circ \qquad \sin\phi = 0.447 \qquad \cos\phi = 0.894$$

$$\text{RING FORCE} = \frac{133\,500\,w}{0.500 \times 2\pi} = 42\,000\,w$$

$$T_1 = \frac{133\,500\,w}{2\pi \times 200 \times 0.447} = 238\,w$$

$$T_2 = \left(w\cos 26.55^\circ - \frac{238\,w}{R_1}\right)R_2 = \left(0.894\,w - \frac{238\,w}{560}\right)448 = 210\,w$$

$$\text{WHERE } R_2 = \frac{200}{0.447} = 448$$

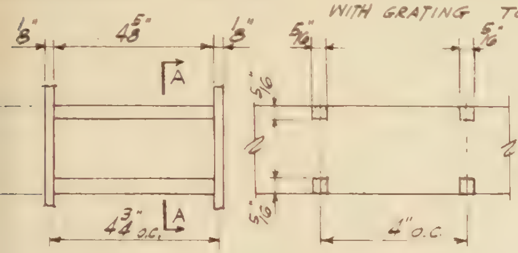
$$R_{1(\phi = k)} = \left[1 + \left(\tfrac{1}{2}\right)^2\right]^{3/2} \times \frac{(400)^2}{8 \times 50} = 560$$

$$K = 0.925 \times \frac{560}{\sqrt{448\,(0.164)}} = 61$$

$$\lambda = 4.89\sqrt{448\,(0.328)} = 58'$$

$$\sigma_{cr} = \frac{5670 \times 10^3}{560} = 10200 \text{ psi}$$

ALCULATION OF δ (4'-0" × 4'-0" SQUARE PANELS WITH GRATING TOP ONLY)



SECTION A-A

TYPICAL STEEL GRATING

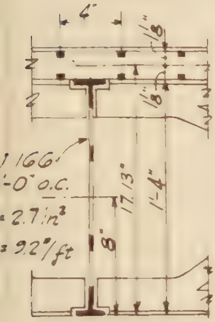
AREA (A-A)

$(\frac{5"}{16})^2 \times 2 \times 3 = 0.59\ in^2/ft$

$0.59 \times 4 = 2.36\ in^2/4\ ft$

$2\frac{1}{4} \times \frac{1}{8} \times \frac{12}{4.75} = 0.70\ in^2/ft$

$0.70 \times 4 = 2.80\ in^2/4\ ft$



$S_{(4'-0")} = 2.36 \times 17.13 + 2.7 \times 8 = 62.1\ in^3$

$A_{(4'-0")} = 2.36 + 2.7 = 5.06\ in^2$

$z = \frac{62.1}{5.06} = 12.2\ in$

$I_{(4'-0")} = 2.36\overset{57.5}{(17.13-12.2)^2} + 116.0 + 2.7(12.2-8.0)^2 = 221\ in^3$

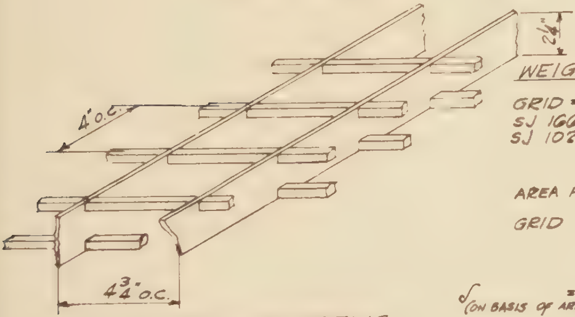
ADDITIONAL I FROM DIAGONAL = $\underline{19}$

$240\ in^3$

$\frac{b\delta^3}{12} = 240\ in^4$

$\delta = \sqrt[3]{\frac{240 \times 12}{48}} = \sqrt[3]{60} = 3.92" = 0.328\ ft$

EQUIVALENT SOLID SHEET
(MINIMUM POSSIBLE STIFFNESS VALUE)



ISOMETRIC OF GRATING

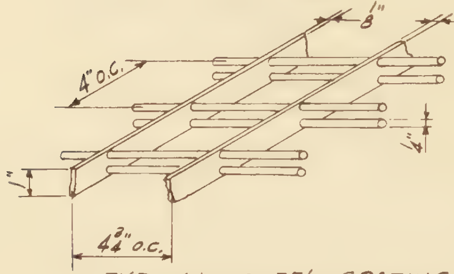
WEIGHT PER SQ. FOOT

GRID = 1.29 × 3.4 = 4.4
SJ 166 = 2 × 9.2/4 = 4.6
SJ 102 = 4.4 × 1.414/4 = $\underline{1.5}$
10.5 PSF

AREA PER SQ.FT. = $\frac{2.7+1.8}{4}$ = 1.11
GRID = $\frac{1.29}{2}$ = $\underline{0.65}$
$1.76\ in^2$

$\delta_{(ON\ BASIS\ OF\ AREA)} = 0.147\ in$

CALCULATION OF δ (3'-9" × 3'-9" - 60° DIAMOND PATTERN, GRATING TOP & BOTTOM)



TYPICAL STEEL GRATING

AREA ROUNDS

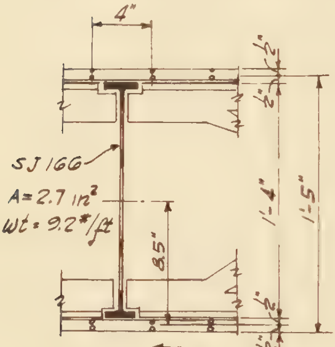
$\pi(\frac{1}{4})^2 \times 2 \times 3 = 0.295 \ in^2/ft$

AREA FLATS

$1" \times \frac{1}{8}" \times \frac{12}{4.75} = 0.315 \ in^2/ft$

$2\,\overline{)\,0.610}$

, AVERAGE = $0.305 \ in^2/ft^2$



SECTION 1-1

AREA GRATING = $0.305 \times 2 \times 3.75 = 2.3 \ in^2/3'\text{-}9"$

$I = 116 + 2.3 \times \overset{166}{\overline{8.5}}^2 = 282 \ in^4$

$\delta_{1-1} = \sqrt[3]{\frac{I \times 12}{b}} = \sqrt[3]{\frac{282 \times 12}{45}} = 4.22"$

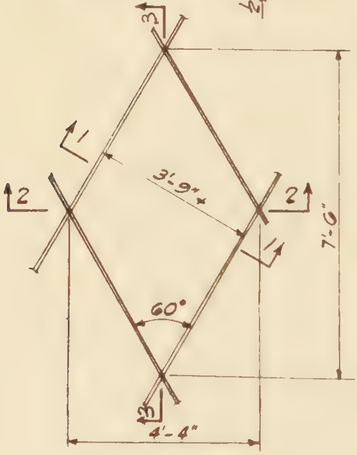
$\delta_{1-1} = \delta_{2-2} = \delta_{3-3}$

SHELL HAS SAME I IN EVERY DIRECTION

WEIGHT

ALUMINIUM SKIN	= 0.45 PSF
GRATING = 2 × 2.34	= 4.68
SJ 166 = $\frac{2 \times 4.33 \times 9.2}{3.75 \times 4.33}$	= 4.90
	10.03 PSF

USE = 10.0 PSF



GRID PATTERN

TABULATION OF FORCES & PROPERTIES

H ft	75'	100'	125'	62½'	50'
L ft	600'	600'	600'	500'	400'
L/H	8.0	6.0	4.8	8.0	8.0
f ft	300'	225'	180'	250'	200'
$\tan\phi_k$	0.500	0.667	0.833	0.500	0.500
$\sin\phi_k$	0.447	0.555	0.640	0.447	0.447
$\cos\phi_k$	0.894	0.832	0.767	0.894	0.894
S_k ft²	302 000	313 000	327 000	209 000	133 500
T_{1_k} k/ft	355 w	300 w	270 w	298 w	238 w
T_{2_k} k/ft	312 w	242 w	200 w	263 w	210 w
RING FORCE	96 000 w	74 800 w	62 200 w	65 500 w	42 000 w
R_{1_k}	838	784	795	700	560
R_{2_k}	670	540	468	560	448
λ ft	72'	64'	59'	65'	58'
δ ft	0.328'	0.328'	0.328'	0.328'	0.328'
$\sigma_{CR(k)}$ psi	6800	7020	7100	8100	10200
K	75	78	86	69	61
PROJECTED AREA (PA) ft²	283 000	283 000	283 000	197 000	126 000
PA/S_k	93.7%	90.5%	86.5%	94.3%	94.3%
DEAD LOAD w = psf	10	10	10	10	10
10#/ft² WIND ≏ w	9.4	9.1	8.7	9.4	9.4

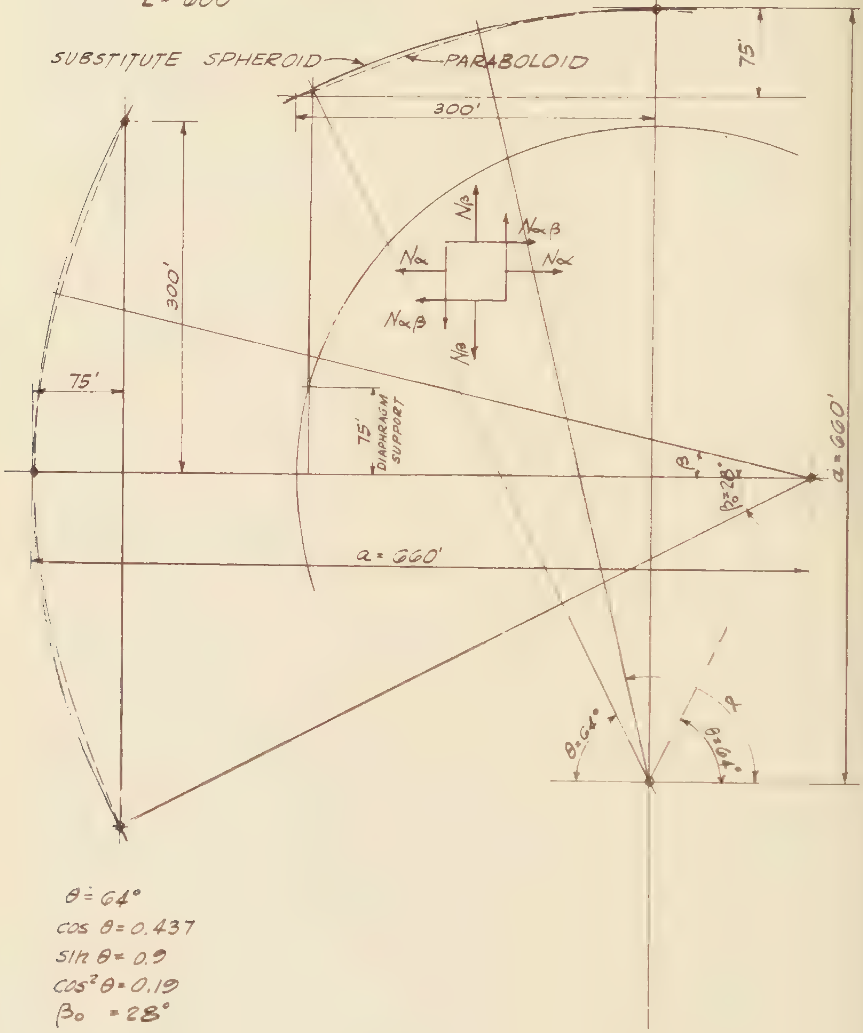
NOTE: 10#/ft² WIND = 45 M.P.H. + 50% SUCTION

CALCULATION OF THE STRESSES IN THE PARABOLOID A

A SHELL SUPPORTED ON TWO DIAPHRAGMS

$H = 75'$ $L/H = 8$

$L = 600'$



$\theta \doteq 64°$

$\cos\theta = 0.437$

$\sin\theta = 0.9$

$\cos^2\theta = 0.19$

$\beta_0 = 28°$

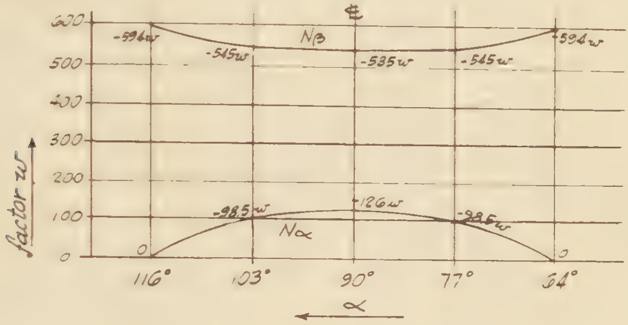
$a = 660'$

1) $N_\alpha = -aw \dfrac{\cos^2\theta - \cos^2\alpha}{\sin^3\alpha} \cos\beta$

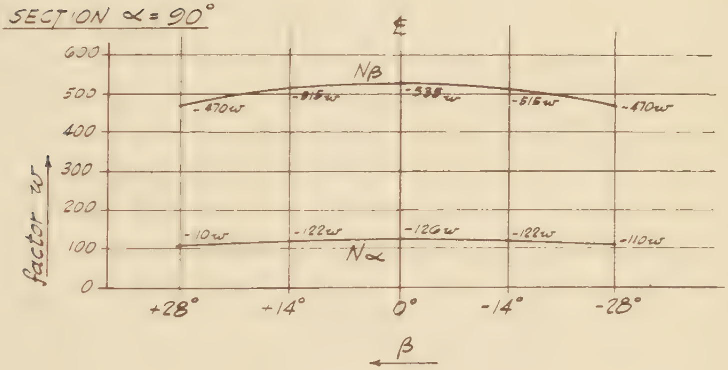
2) $N_\beta = aw\left[\dfrac{\cos^2\theta - \cos^2\alpha}{\sin^3\alpha} - \sin\alpha\right]\cos\beta$

3) $N_{\alpha\beta} = aw \dfrac{\cos\alpha}{\sin^3\alpha}\left[1 + \sin^2\alpha - \cos^2\theta\right]\sin\beta$

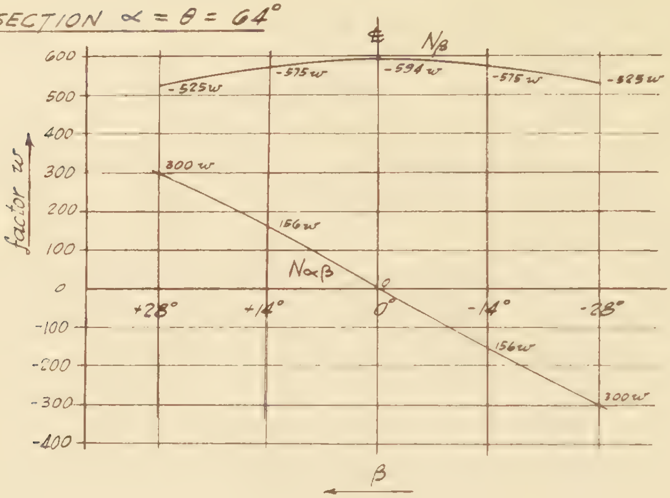
A) SECTION β=0°



B) SECTION α = 90°



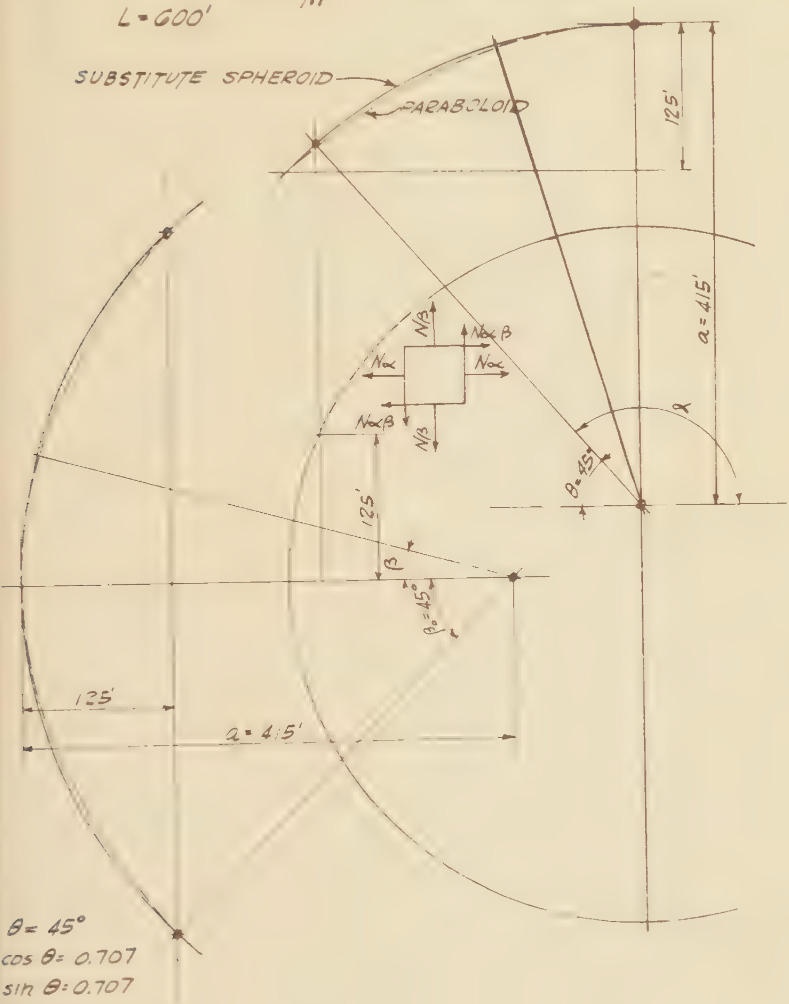
c) SECTION $\alpha = \theta = 64°$



CALCULATION OF THE STRESSES IN THE PARABOLOID AS A SHELL SUPPORTED ON TWO DIAPHRAGMS

$h = 125'$ $L/H = 4.8$

$L = 600'$



$\theta = 45°$

$\cos\theta = 0.707$

$\sin\theta = 0.707$

$\cos^2\theta = 0.5$

$\beta_0 = 45°$

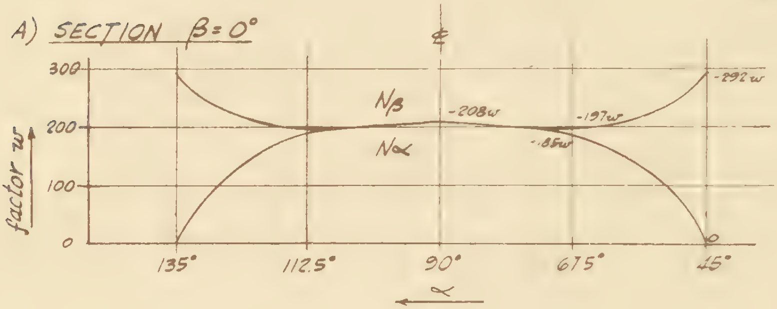
$a = 415'$

1) $N_\alpha = 415w\,\dfrac{(0.5-\cos^2\alpha)}{\sin^3\alpha}\,\cos\beta$

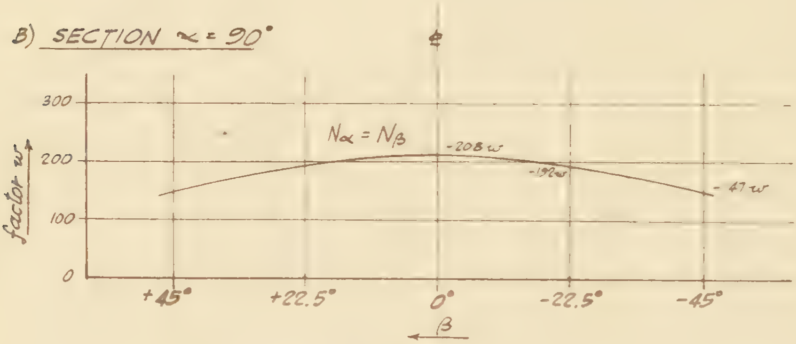
2) $N_\beta = 415w\left[\dfrac{0.5-\cos^2\alpha}{\sin^3\alpha} - \sin\alpha\right]\cos\beta$

3) $N_{\alpha\beta} = 415w\,\dfrac{\cos\alpha}{\sin^3\alpha}\left[0.5+\sin^2\alpha\right]\sin\beta$

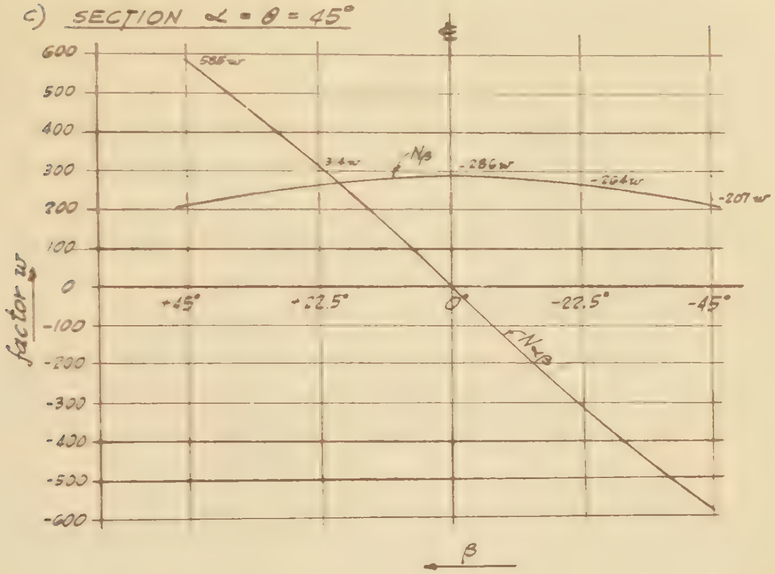
A) SECTION β = 0°



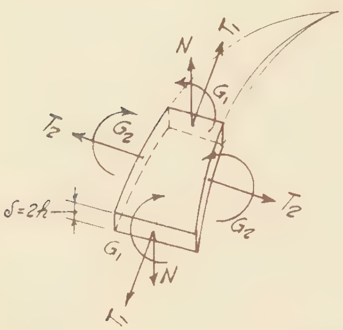
B) SECTION α = 90°

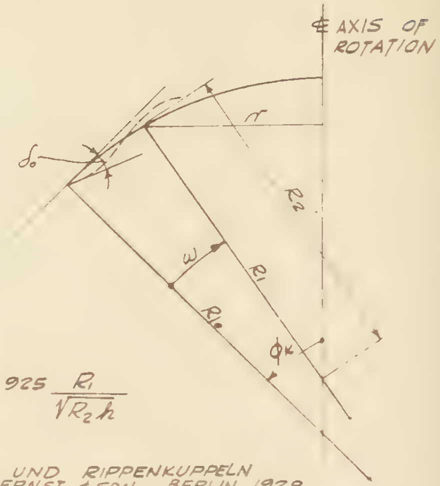


c) SECTION $\alpha = \theta = 45°$



BENDING IN SHELLS





SHELL CONSTANT $k = 0.925 \dfrac{R_1}{\sqrt{R_2 h}}$

FROM DISCHINGER[2]: SCHALEN UND RIPPENKUPPELN
WILHELM ERNST & SON BERLIN 1928

Page 228 (97 a) $N = 2h\tau = \dfrac{h^2}{R_2} \cdot C e^{-k\omega} \cdot \cos(k\omega + \psi)$

(97 b) $T_1 = 2h\sigma_1 = \dfrac{h^2}{R_2} \cdot \operatorname{ctg}\phi \, C e^{-k\omega} \cdot \cos(k\omega + \psi)$

(97 c) $T_2 = 2h\sigma_2 = \dfrac{h^2}{R_1} \cdot k\sqrt{2} \, C e^{-k\omega} \cdot \sin\left(k\omega + \psi + \dfrac{\pi}{4}\right)$

(97 d) $\delta = -\sqrt{\dfrac{3}{4}(1-\nu^2)} \cdot \dfrac{1}{E} \cdot C e^{-k\omega} \cdot \sin(k\omega + \psi)$

(97 e) $G_1 = \sigma_{B_1} \cdot 4 \cdot \dfrac{h^2}{6} = \dfrac{-h^3}{R_1\sqrt{2}\sqrt{\dfrac{3}{4}(1-\nu^2)}} \cdot k \, C e^{-k\omega} \cdot \cos\left(k\omega + \psi + \dfrac{\pi}{4}\right)$

(97 f) $G_2 = \sigma_{B_1} \cdot 4 \cdot \dfrac{h^2}{6} = \dfrac{2h^3 E}{3(1-\nu^2)} \cdot \dfrac{\operatorname{ctg}\phi}{R_2} \cdot \delta + \nu G_1$

FOR SHELL FULLY RESTRAINED, $\psi = 0$

Page 230 (101 a) $\tau = (\sigma_2)_0 \cdot \dfrac{1}{k} \cdot e^{-k\omega} \cdot \cos k\omega$

(101 b) $\sigma_1 = (\sigma_2)_0 \cdot \dfrac{\operatorname{ctg}\phi}{k} \cdot e^{-k\omega} \cdot \cos k\omega$

(101 c) $\sigma_2 = (\sigma_2)_0 \sqrt{2}\ e^{-k\omega} . \sin\left(k\omega + \frac{\pi}{4}\right)$

(101 d) $\delta = \frac{(\sigma_2)_0}{E} . 2ke^{-k\omega} . \sin k\omega$

(101 e) $\sigma_{B_1} = (\sigma_2)_0 \sqrt{\frac{6}{1-\nu^2}} . e^{-k\omega} . \cos\left(k\omega + \frac{\pi}{4}\right)$

(101 f) $\sigma_{B_2} = \nu . \sigma_B - \frac{hE . ctg\,\phi}{(1-\nu^2)R} . \delta$

FOR SHELL, HINGED AT SUPPORT $\psi = \frac{\pi}{4}$

Page 231 (102 a) $\tau = (\sigma_2)_0 \cdot \frac{1}{k\sqrt{2}} . e^{-k\omega} . \cos\left(k\omega + \frac{\pi}{4}\right)$

(102 b) $\sigma_1 = (\sigma_2)_0 \cdot \frac{ctg\,\phi}{k\sqrt{2}} . e^{-k\omega} . \cos\left(k\omega + \frac{\pi}{4}\right)$

(102 c) $\sigma_2 = (\sigma_2)_0\ e^{-k\omega} . \cos k\omega$

(102 d) $\delta = \frac{(\sigma_2)_0}{E} . k\sqrt{2} . e^{-k\omega} . \sin\left(k\omega + \frac{\pi}{4}\right)$

(102 e) $\sigma_{B_1} = (\sigma_2)_0 \sqrt{\frac{3}{1-\nu^2}} . e^{-k\omega} . \sin k\omega$

(102 f) $\sigma_{B_2} = \nu\sigma_{B_1} - \frac{hE . ctg\,\phi}{(1-\nu^2)R} . \delta$

DOME WITH 75' RISE

PROPERTIES

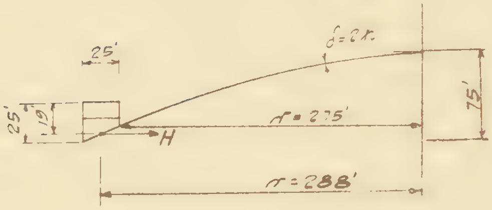
$k = 75$ (SHEET 3)

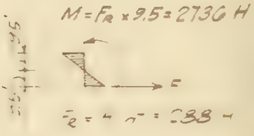
$R_2 = 670'$

$\tan\phi = 0.5 \qquad \sin^2\phi = 0.2$

$A = 1.27\ ^{in^2}/ft$

$\delta/2 = 0.164\ ft = h$





$$H = 1 \text{----} \quad \Delta r_{0(s)} \cdot E = \sin^2\phi \frac{k R_2}{h} = \frac{0.2 \times 75 \times 670}{0.158} = 63600$$

RING

PROPERTIES

$A = 144\ in^2 = 1\ ft^2$

$I = 0.5 \times 9.5^2 \times 2 = 90\ ft^4$

$S = 9.5\ ft^3$

$$\sigma_{20(r)} = \frac{288}{1} + \frac{2736}{90} \times 9.5 = 288 + \frac{2736}{9.5} = 580$$

$$E(\Delta r)_R = 580 \times 288 = 167040$$

CALCULATION FOR $\Delta t = \pm 20°$ TEMPERATURE DIFFERENCE BETWEEN SHELL AND RING

$$E\,\Delta r = 20 \times \underset{\alpha}{6 \times 10^{-6}} \times \underset{E}{4.32 \times 10^{6}} \times 288 = 530 \times 288 = 152000$$

$$= E(\varepsilon_2)_0\, r$$

$$x(63600 + 167040) = 152000$$

$$H = x.1 = 0.65$$

$$\Delta r_{0(s)} E = 63600 \times 0.65 = 41000 = \sigma_{20}\, r$$

$$\sigma_{20} = \frac{41000}{288} = 143\ ^{k}/ft^2 = 1000\ psi$$

ADDITIONAL RING FORCE $F_R = 0.65 \times 288 = 190^{k}$

a) DOME FULLY RESTRAINED AT RING

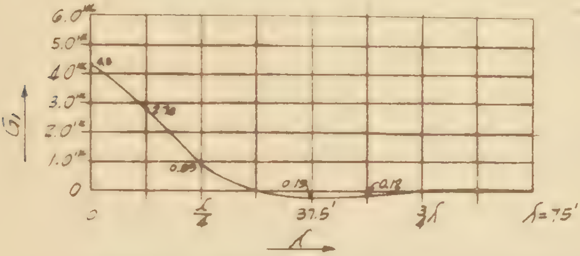
$$\sigma_{b_1} = 2.45 \times 144 \times 0.707 = 350\ ^{k}/_{ft^2} \times 0.707 = 246\ ^{k}/_{ft^2}$$

$$G_{1(0)} = \sigma_{b_1(0)} \cdot \frac{2}{3} k^2 = 0.164^2 \times 0.667 \times 246 = 0.0167 \times 246 = 4.3^{1k}$$

b) DOME HINGED AT RING

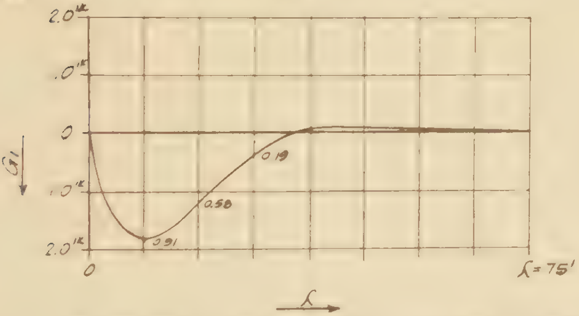
$$\text{ROTATION } \theta = \frac{143 \times 75 \times 1.414 \times 0.707}{4.32 \times 10^6} = \frac{10800}{4.32 \times 10^6} = 2500 \times 10^{-6} \text{ RADIANS}$$

$$= 8.6 \text{ MINUTES}$$



G_1 = MOMENT LINE FOR FULLY RESTRAINED CONDITION

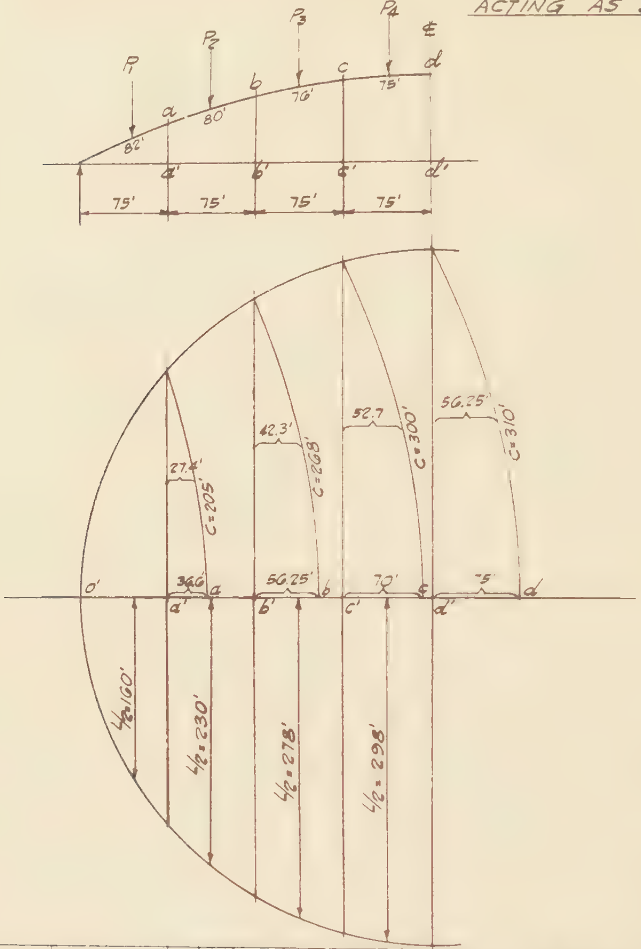
1" = 4^1k



G_1 = MOMENT LINE FOR FULLY HINGED CONDITION

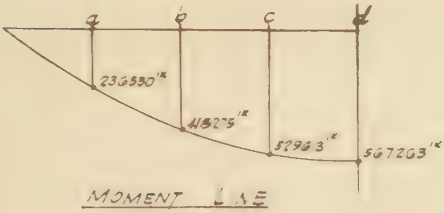
1" = 2^1k

CALCULATION OF BENDING MOMENTS OF SHELL SECTIONS ACTING AS BEAMS.

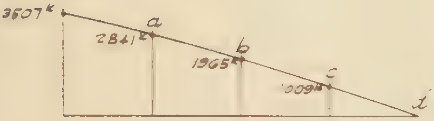


SECTION		AREA (FT²)	P D.L. (KIPS) @ 10 PSF	W = D.L. + LL @ 20 PSF	RING WT @ 700# FT		ΣP S(D.L.+LL) + P R	
S1	320 × 82	26 240	263^K	526^K	100×0.7×2=140^K		P1=	666^K
S2	460 × 80	36 800	368	736	100×0.7×2=140		P2=	876
S3	556 × 76	42 256	423	846	82×0.7×2=115		P3=	961
S4	596 × 75	44 700	447	894	78×0.7×2=110		P4=	1004
		149996	1501	3002	360	505		R=3507^K

SHEET 20



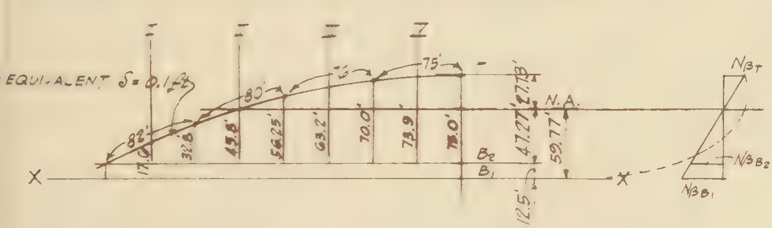
MOMENT LINE



SHEAR LINE

* EQUIVALENT δ TAKEN TO
NDICATE MEMBRANE STRESSES
AS COMPUTED FOR THE SHELL.

MOMENT OF INERTIA AND SECTION MODULUS



	AREA	y	Ay	y^2	$Ay^2 = I_{x-x}$
RING	1.4	6.25	8.75	39.06	54.7
I	8.40	30.1	253.0	906.01	7615
II	8.17	58.3	478.0	3399	27867
III	8.07	75.7	610.0	5730	46177
IV	7.75	86.4	670.0	7465	57888
	33.79		2019.75		139602

$$z = \frac{2019.75}{33.79} = 59.77'$$

$$I_{n-n} = I_{x-x} - Az^2 = 139602 - 33.79 \times 3572.5 = 18888 \times 2 = 37776 \text{ ft}^4$$

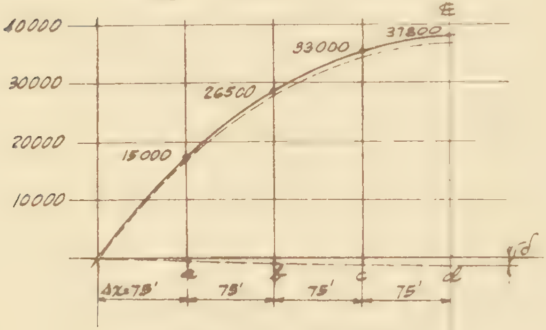
$$S_T = \frac{37776}{27.73} = 1360\ ft^3$$

$$S_{B_1} = \frac{37776}{59.77} = 630\ ft^3$$

$$f_T = \frac{M}{S_T} = \frac{567263}{1360} = 420\ {}^{K}/ft^2 = 2900\ psi$$

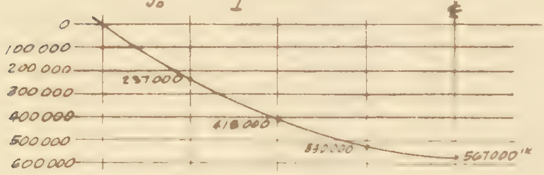
$$f_{B_1} = \frac{M}{S_{B_1}} = \frac{567263}{630} = 900\ {}^{K}/ft^2 = 6250\ psi$$

$$f_{B_2} = \frac{M}{S_{B_2}} = \frac{567263}{800} = 710\ {}^{K}/ft^2 = 4950\ psi$$

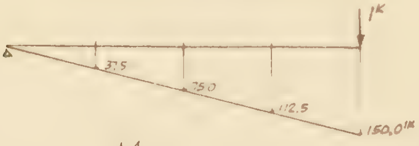


MOMENT OF INERTIA

$$E\delta = 2\int_0^d \frac{M_1 M_0\, ds}{I} = 2\Delta x \sum_0^d M_1 M_0$$



M_0 (BENDING MOMENT)



M_1

CTION	$M_0 M_1$	I	$\frac{M_0 M_1}{I}$
0	22×10^6	7500	295
a	8.9×10^6	15000	600
b	31.4×10^6	26500	1190
c	59.5×10^6	33000	1800
d	85.0×10^6	37800	2250
			6135

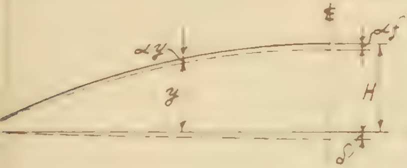
$$\underline{1125} \ (\tfrac{1}{2} d \text{ FOR CORRECTION OF INTEGRATION})$$

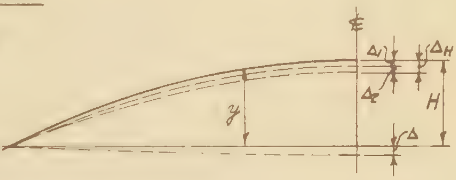
$$\Sigma \frac{M_0 M_1}{I} = 5010$$

$$E\Delta_1 = 2 \times 5010 \times 75 = 756000$$

$$\Delta_1 = \frac{2\Delta x \sum_0^d M_1 M_0}{E} = \frac{756000 \times 12}{4.17 \times 10^6} = \frac{9.7 \times 10^6}{4.17 \times 10^6} = 2.26''$$

EFLECTION Δ_1 FOR 75 ft RISE





EFLECTION Δ_H $\quad$ Δ_1 FROM BEAM ACTION

$\Delta_H = \Delta_1 + \Delta_2$ $\quad$ Δ_2 FROM MEMBRANE STRESSES $= \frac{f}{E} y$

= MAX DEFLECTION AT CENTER FROM LOADS (D.L.+L.L.) DUE TO BEAM ACTION = 2.26"

= " " " " " MEMBRANE STRESSES $= \frac{2 \times 6000_{\text{PSL}} \times 75' \times 12}{29 \times 10^6} = 0.37$

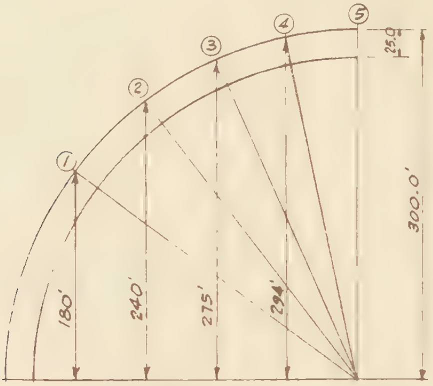
$2.63''$

$\Sigma \approx 3''$

SHELL IN HORIZONTAL POSITION

(RING & SHELL ACTING TOGETHER)

1) DEFLECTION DUE TO BEAM ACTION = Δ_1



PLAN

	①	②	③	④	⑤
RIZE ft	27'	48'	63'	72'	75'
I ft⁴	8880	7000	12560	13080	13600

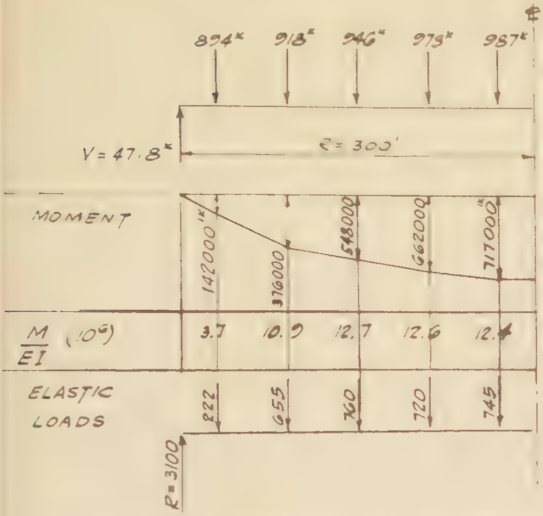
SKIN = 9.0 PSF

δ = EQUIVALENT THICKNESS $\frac{9.0}{40.8} = 0.22''$

(NOTE: MOMENT OF INERTIA WAS CALC. ON ASSUMPTION OF δ EQUIV. THICK.)

LOADS

						℄
D.L. OF SHELL @ 10 PSF	116ᵏ	279	332	360	373ᵏ	
D.L. @ 10 PSF + L.L. @ 10 PSF	332ᵏ	558	664	720	746ᵏ	
RING @ 2.0 ᵏ/ft	562ᵏ	360	282	253	241ᵏ	
TOTAL	894	918	946	973	987ᵏ	
DISTANCE 0'	30'	90'	150'	210'	270'	300'



Δ_1 ℄ $(10^6) = 3100 \times 300 - 222 \times 270 - 655 \times 210 - 760 \times 150 - 720 \times 90 - 745 \times 30 = 531\,000$

Δ_1 ℄ $= 0.53$ ft $= 6.35''$

STRESSES AT ℄

$$f = \left(\frac{Mc}{I}\right)_{TOP} = \left(\frac{Mc}{I}\right)_{CROWN} = \frac{717\,000 \times 29}{13600 \times (12)^2} = 10700 \text{ PSI}$$

2) DEFLECTION DUE TO MEMBRANE STRESSES = Δ_2

$$\Delta_2 = \frac{2 \times 6000 \times 75 \times 12}{30 \times 10^6} = 0.37''$$

$$\Delta = \Delta_1 + \Delta_2 = 6.35 + 0.37 = 6.72''$$

FROM SHEET 11 AVERAGE NORMAL STRESS AT CROWN

$$= \frac{535\,w + 126\,w}{2} = 330.5\,w$$

AT 20 PSF TOTAL LOAD $f_{CROWN} = 330.5 \times 20 = 6610$ PSI

CORRECTION FACTOR $\mu = \frac{6610}{10700} = 0.618$

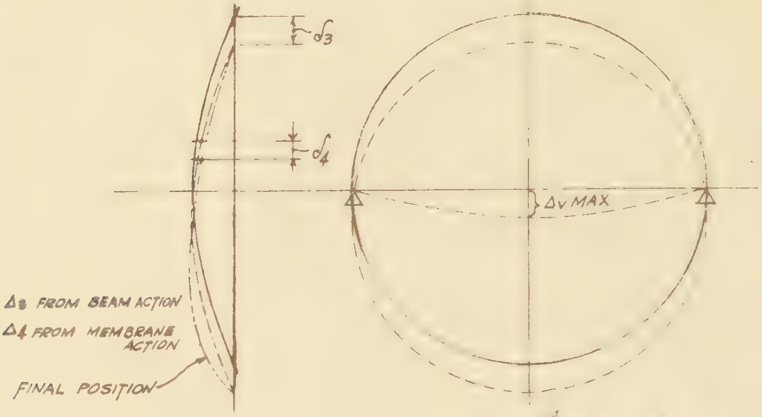
Δ_1 CORRECTED $= 6.35 \times 0.618 = 3.92$

$\Delta_2 \qquad = 0.37$

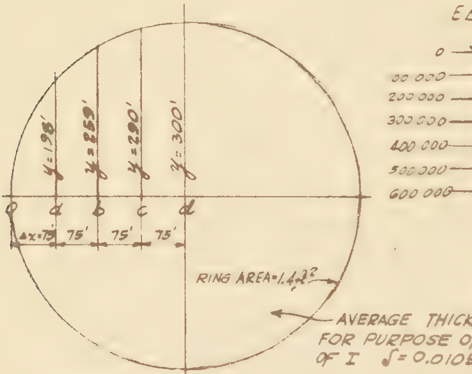
Δ TOTAL $= \Delta_{LL+DL} \qquad = 4.29''$

$= 2.96''(\Delta_{D.L.}) + 1.33''(\Delta_{L.L.})$

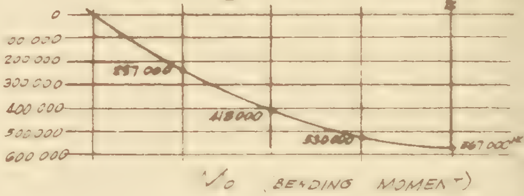
DEFLECTION Δv OF SHELL IN UPRIGHT POSITION

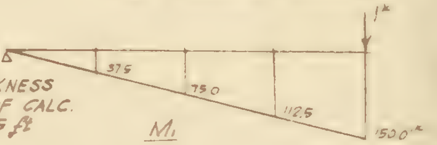


$$E\Delta = 2\int_0^d \frac{M_1 M_0\, ds}{I} = 2\Delta x \sum_0^d M_1 M_0$$



ORDINATES OF DISK





SECTION	$M_0 M_1$	CALC OF I	I *	$\frac{M_0 M_1}{I}$
0	0	0	0	0
a	8.9×10^6	$0.007 \times 1.98^3 \times 10^6$	0.54×10^5	164
b	31.4×10^6	$0.007 \times 2.59^3 \times 10^6$	1.22×10^5	257
c	59.5×10^6	$0.007 \times 2.90^3 \times 10^6$	1.72×10^5	334
d	85.0×10^6	$0.007 \times 3.00^3 \times 10^6$	1.90×10^5	473
				1228
				−236
			$\frac{\Sigma M_0 M_1}{I} =$	992

(½d FOR CORRECTION OF INTERGATION)

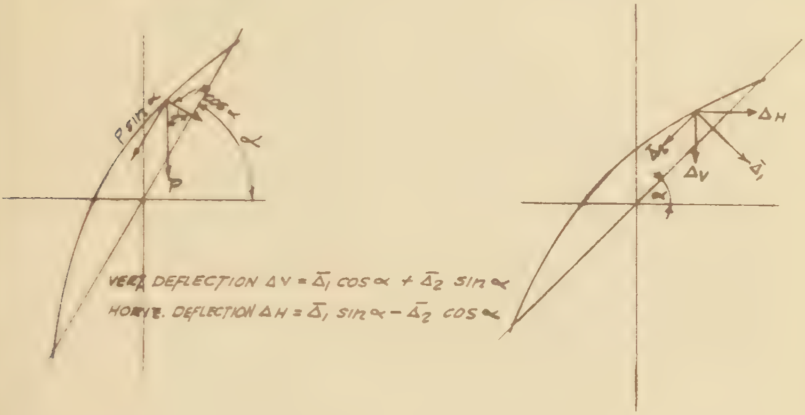
* SHELL ONLY (i.e WITHOUT RING)

$E\Delta_3 = 2 \times 992 \times 75 = 148000$

$\Delta_3 = \frac{148000 \times 12}{4.32 \times 10^6} = \frac{1780000}{4.32 \times 10^6} = 0.41''$ FOR FULL DL. + L.L.

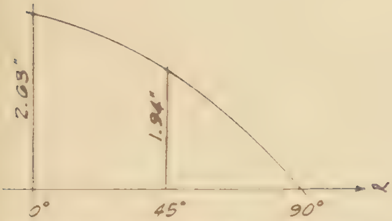
0.24" FOR D.L. ONLY

$\Delta_4 =$ NEGLIGIBLE



VERT. DEFLECTION $\Delta V = \bar{\Delta}_1 \cos\alpha + \bar{\Delta}_2 \sin\alpha$

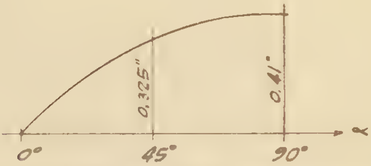
HORIZ. DEFLECTION $\Delta H = \bar{\Delta}_1 \sin\alpha - \bar{\Delta}_2 \cos\alpha$



Δ_H DEFLECTION FOR D.L. + L.L. = 20 PSF

$\Delta_H = \Delta_{1\,MAX}\, 0.57 \cos\alpha + \Delta_{1\,MAX}\, 0.43 \cos^2\alpha$

$= 1.50 \cos\alpha + 1.13 \cos^2\alpha$

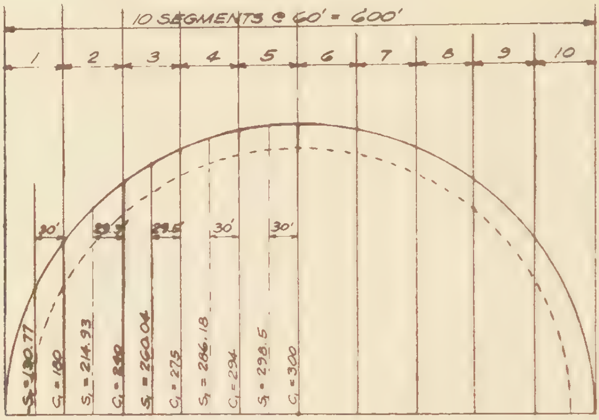


Δ_V = DEFLECTION FOR D.L. + L.L. = 20 PSF

$\Delta_V = 0.24 \sin\alpha + 0.17 \sin^2\alpha$

RING & SHELL IN VERTICAL POSITION (ACTING TOGETHER)

DIMENSIONS & PROPERTIES OF RING & SHELL

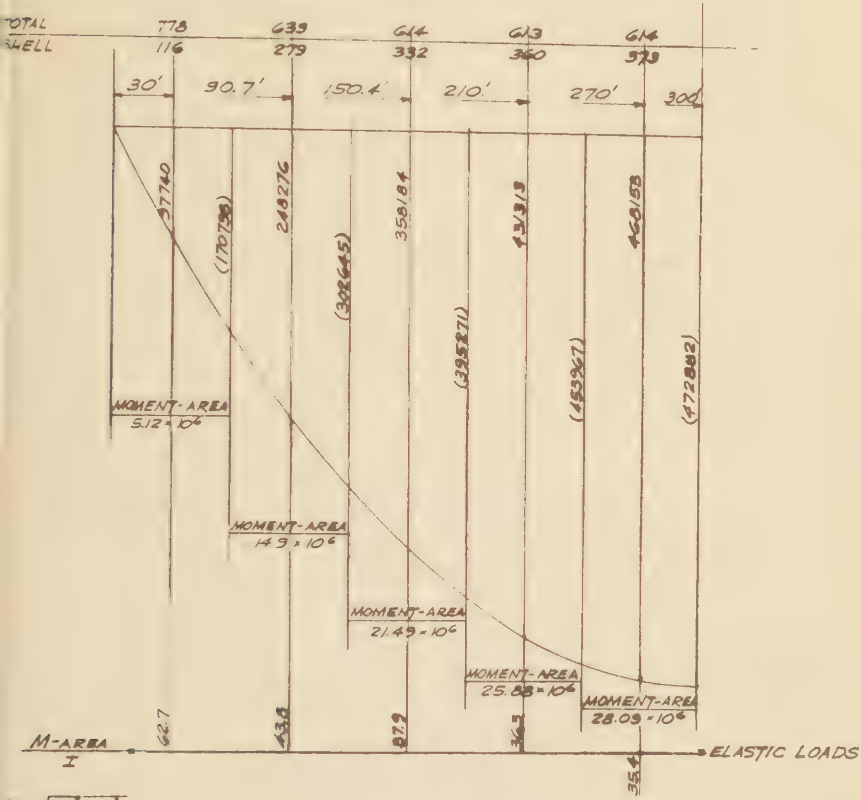


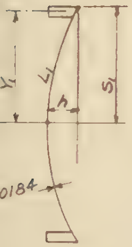
WEIGHTS OF RING & SHELL

SEGMENT	RING	SHELL	TOTAL
1	662^K	116^K	778^K
2	360	279	639
3	282	332	614
4	253	360	613
5	241	373	614

1. BEAM ACTION

MOMENT DIAGRAM OF WEIGHTS OF RING TRUSS AND SHELL APPLIED AS LOADS

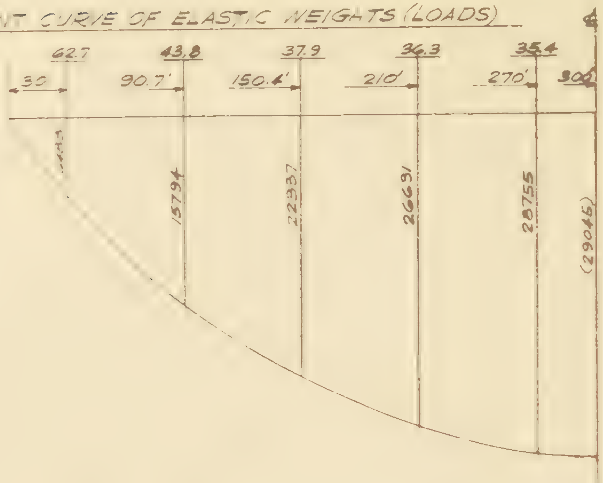




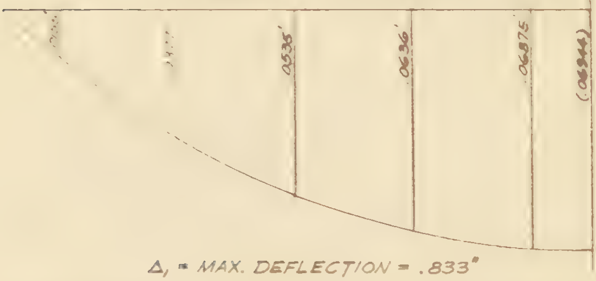
MOMENTS OF INERTIA AT CENTERS OF GRAVITY OF SEGMENTS

SEGMENT	Y1	S1	L1	I OF RING	I OF SHELL	TOTAL I	h
1	98.775	130.77	1318	54100 FT.⁴	27500 FT⁴	81600 FT.⁴	14.25
2	196.945	214.93	219.45	215800	124100	339900	38.6
3	239.13	260.04	267.95	318500	222000	540500	56.4
4	273.05	286.18	296.68	415000	298500	713500	68.25
5	285.93	298.5	310.36	455000	339000	794000	74.25

MOMENT CURVE OF ELASTIC WEIGHTS (LOADS)



DEFLECTION CURVE FOR BEAM ACTION

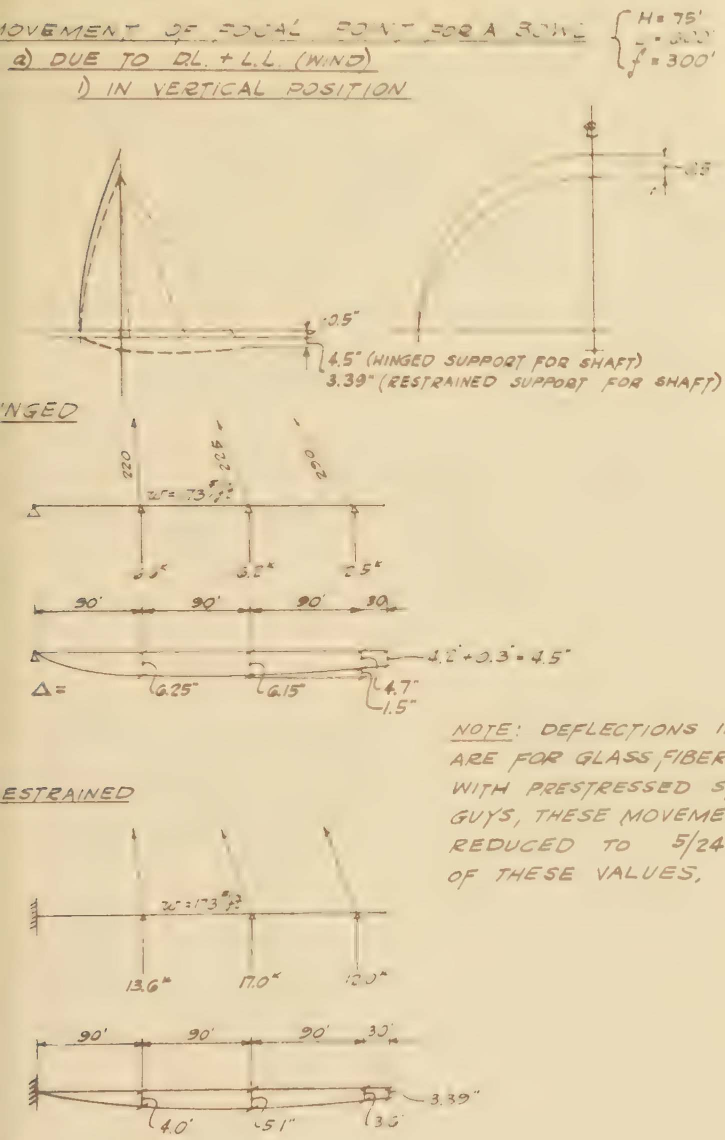


Δ_1 = MAX. DEFLECTION = .833"

Δ_2 = DEFLECTION DUE TO MEMBRANE STRESSES

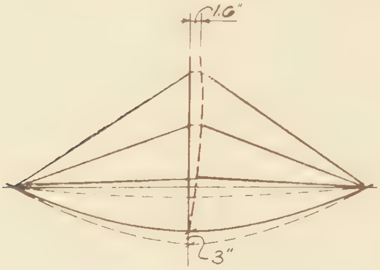
$$= \frac{f \times l}{E} = \frac{3550}{30 \times 10^6} \times 312 \times 12 = 0.44''$$

FOR VALUE OF "f" SEE SHEET #9



NOTE: DEFLECTIONS INDICATED ARE FOR GLASS FIBER CABLES. WITH PRESTRESSED STEEL CABLE GUYS, THESE MOVEMENTS ARE REDUCED TO 5/24 OR 20% OF THESE VALUES.

2) DEFLECTIONS OF BOWL IN HORIZONTAL POSITION



MOVEMENT OF FOCAL POINT DUE TO CHANGE OF SHAPE, CAUSED BY SHORTENING OR LENGTHENING FROM MEMBRANE FORCES AND TEMPERATURE CHANGES.

$$y^2 = 4fx$$

$$f = \frac{y^2}{4x} = \frac{L^2}{4 \times 4H} = \frac{L^2}{16H}$$

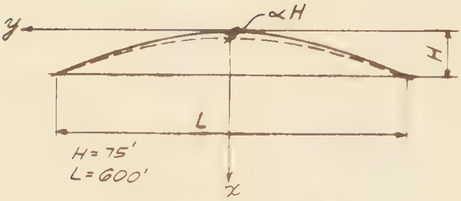
WHERE H IS VARIABLE

$$\frac{df}{dH} = \frac{L^2}{16} \times -H^{-2}$$

$$df = \frac{-L^2}{16} \times \frac{dH}{H^2}$$

$$= \frac{-36 \times 10^4 \times dH}{16 \times 0.5625 \times 10^4} = \frac{-36\,dH}{9}$$

$$= -4\,dH$$



dH FOR 40° TEMP. CHANGE

$$= -75 \times 6 \times 10^{-6} \times 40^\circ \times 12 \text{ in/ft}$$

$$= -21.6 \times 10^{-2}$$

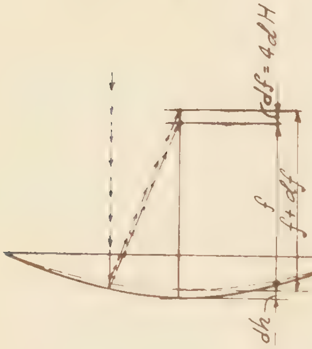
$$df = +86.4 \times 10^{-2} = 0.86 \text{ in}$$

FROM MEMBRANE STRESSES $\sigma = 6000$ psi

$$dH = \frac{6000}{3 \times 10^7} \times 75 \times 12 = \frac{5.4 \times 10^6}{3 \times 10^7}$$

$$= 0.18 \text{ in}$$

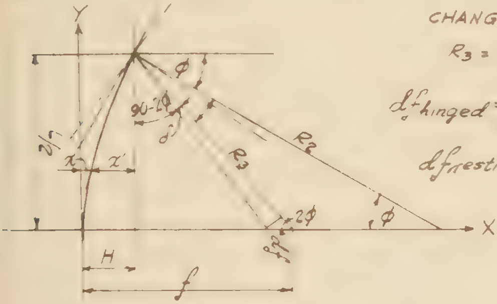
$$df = 4\,dH = 0.72 \text{ in}$$



b) MOVEMENT OF FOCAL POINT DUE TO DISTORTION
DIFFERENCES IN STRAIN AT INTERSECTION OF RING AND SHELL
BROUGHT ABOUT BY A TEMPERATURE GRADIENT AND LOADS
CAUSING DISTORTIONS OF THE SHELL

1) SHELL IN VERTICAL POSITION

FOR 20° TEMP. DIFFERENCE, DISTORTION $\delta_T = 2.5\times10^{-3}$
(SHELL ASSUMED TO BE HINGED AT RING) FROM LOAD $\delta_L = \delta_T$
CAN BE NEGLECTED.



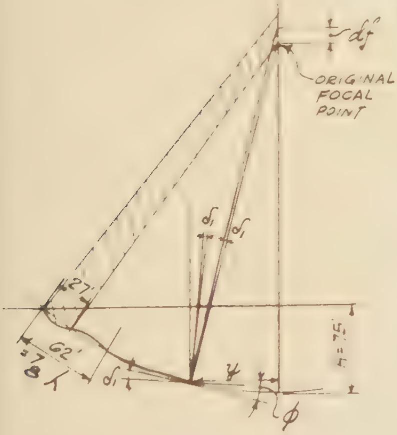
CHANGE IN FOCAL LENGTH

$$R_3 = \sqrt{\left(\tfrac{L}{2}\right)^2+(f-H)^2} = \sqrt{(300)^2+(225)^2} = 375$$

$$df_{hinged} = \frac{R_3\delta}{\sin 2\phi} = \frac{375\times2.5\times10^{-3}}{0.8} = 1.17\ ft$$

$$df_{restrained} = 0.6\, df_{hinged} = 0.702\ ft$$

2) SHELL IN HORIZONTAL POSITION



df_{hinged} ∮ 20° TEMP. VARIATION $\delta = 2.5\times10^{-3}$
FROM LIVE LOAD $= 1.25\times10^{-3}$
$\Sigma\delta = 3.75\times10^{-3}$

SINCE GENERALLY $df = \frac{R_3\delta}{\sin 2\phi}$; $R_3 = \frac{\psi}{\sin 2\phi}$

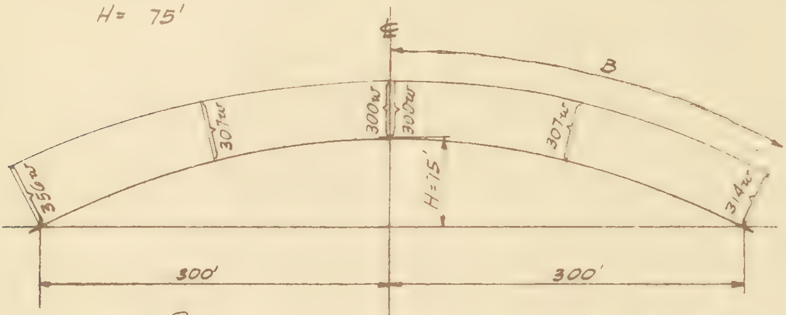
$$df_{hinged} = \frac{R_3\delta}{\sin 2\phi} = \frac{375\times3.75\times10^{-3}}{0.8} = 1.8\ ft$$

$$df_{restrained} = 0.6\times1.8 = 1.08\ ft$$

MEMBRANE FORCES IN DOME SHELL, SUPPORTED ALONG CIRCUMFERENCE

a) SYMMETRICAL LOADS

$L = 600'$
$H = 75'$

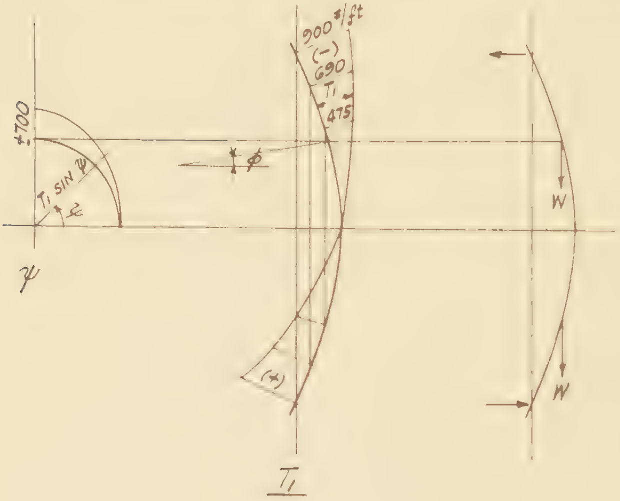


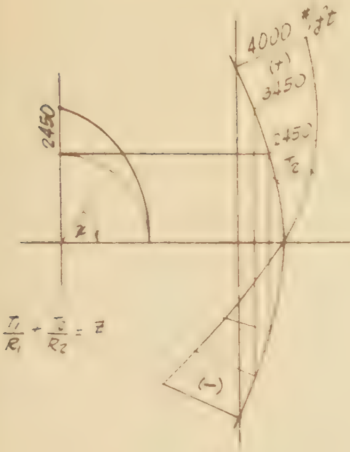
$T_1 = \dfrac{Q}{2\pi r \sin\phi}$ $T_2 = \left(w \cos\phi - \dfrac{T_1}{R_1}\right) R_2$

RING FORCE $F_R = \int T_2\, ds = B T_2 w = 312 \times 310\, w = 1930^k$
WHERE $w = 20$ PSF

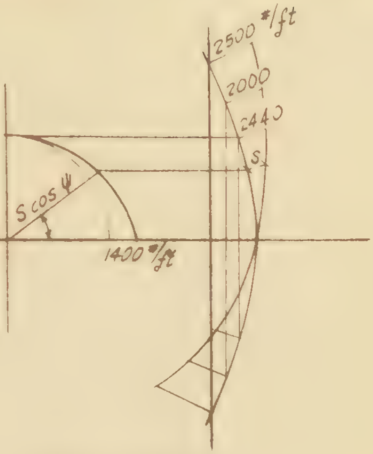
b) ANTISYMMETRICAL LOADS (SHELL IN VERTICAL POSITION)

$W = 10^{\#}$



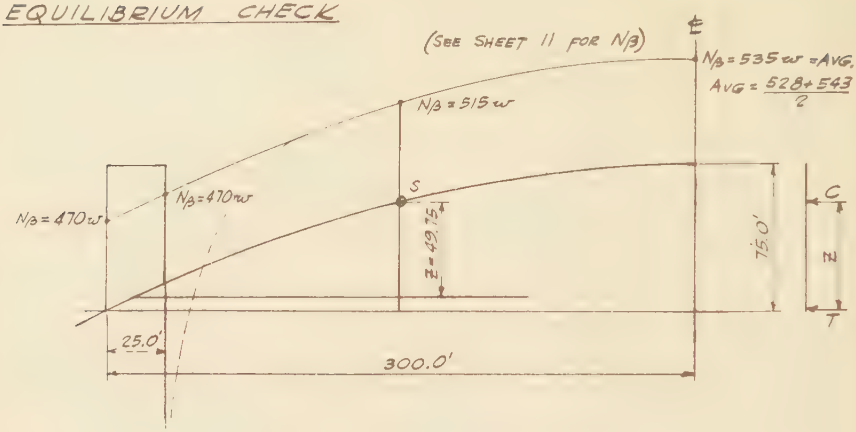


T_2



SHEAR FORCE S

EQUILIBRIUM CHECK



W_T (D.L. + L.L. + RING) = $3507^K \times 2$ SEE SHEET 20
WEIGHT $\left(\underset{\text{SHELL ONLY}}{\text{D.L.} + \text{L.L}}\right)$ = $3002^K \times 2$ 86% OF TOTAL LOAD
M_T = $567000^{\prime K}$ INCLUDES 700#/ft RING
MOMENT $\left(\underset{\text{SHELL ONLY}}{\text{D.L.} + \text{L.L.}}\right)$ = $485000^{\prime K}$

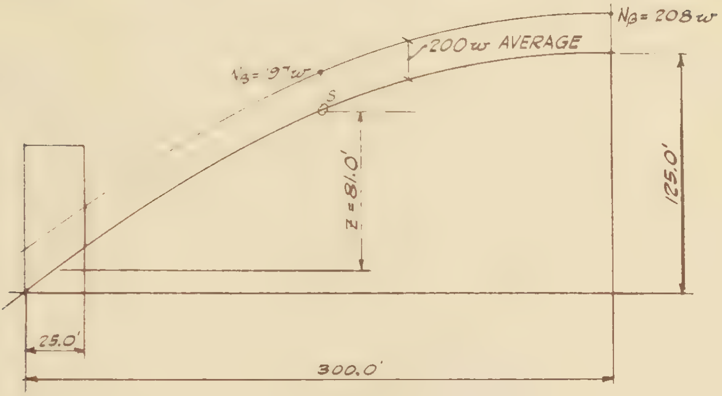
$\Sigma N_\beta ds\ z = 143000\, w \times 49.75 \times 2 = M$
WHERE: $w = 20$ PSF
$N_\beta = 10000^{\#}$
$\left(\sigma = \frac{10000}{1.76} = 5700^{\#}\right)$

$M = 2860^K \times 49.75 \times 2 = 282000^{\prime K}$ OF FULL SECTION AT ℄
$M = 0.57\, M_{(\text{D.L.} + \text{L.L. OF SHELL ONLY})}$

ABOUT 43% OF THE SHELL LOAD HAS TO BE CARRIED BY THE RING.

NOTE: RING IS DESIGNED FOR 50% SHELL LOAD UNIFORMLY DISTRIBUTED OVER PERIMETER.

EQUILIBRIUM CHECK



$M_T = 525\,000'^{K}$

$\Sigma N_\beta ds\, z = 65000 \times w \times 81 \times 2 = M$

$w \times 200 = 10\,000^{\#}$

$w = 50$ PSF

$M = 3250^{K} \times 162 = 520\,000'^{K}$

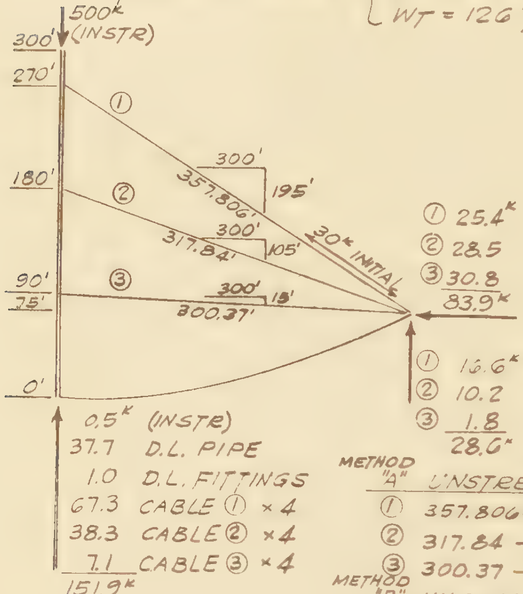
$\therefore$ THE FULL SHELL WEIGHT + $10^{\#}$LL CAN BE CARRIED BY THE SHELL.

GUY CABLE ANALYSIS

FOCAL TOWER VERTICAL
DEAD LOAD ONLY - NO WIND
INITIAL STRESS = 30^{k}

CABLES - GLASS FIBER { 1.6" ⌀, WT = 1.8 #/FOOT
AREA = 2 □", E = 5×10^6

FOCAL TOWER { 24" O.D. PIPE, ½" WALLS
WT = 126 #/FOOT, I = 2580 IN^4



METHOD "A" UNSTRETCHED CABLE LENGTHS

① $357.806 - \frac{30 \times 10^3 \times 357.806}{10^7} = 356.733$

② $317.84 - 0.95 = 316.89$

③ $300.37 - 0.90 = 299.47$

METHOD "B" UNSTRETCHED CABLE LENGTHS

① $n = \frac{W}{8H} = \frac{0.540}{8(30)} = 0.00225$

$\text{CABLE WT} = 1.8 \times 357.8 \times \frac{300.0}{357.8} = 540^{\#}$

$L' = \ell\left(1 + \frac{8}{3}n^2\right)$ WHERE $\ell = 357.806'$

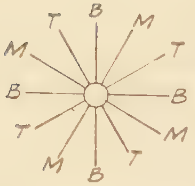
$= 357.806\,(1 + 0.0000135) = 357.811'$

$L' = L' - \frac{H\ell}{AE}\left(1 + \frac{16}{3}n^2\right)$

$= 357.811 - \frac{30 \times 10^3\,(357.806)}{10^7} \times (1 + 0.000027)$

$= 356.741$

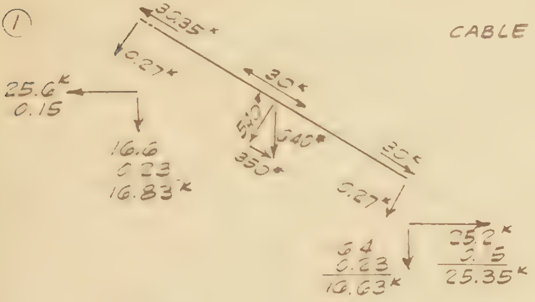
∴ UNSTRETCHED LENGTH BY METHOD "A" SUFFICIENTLY ACCURATE.



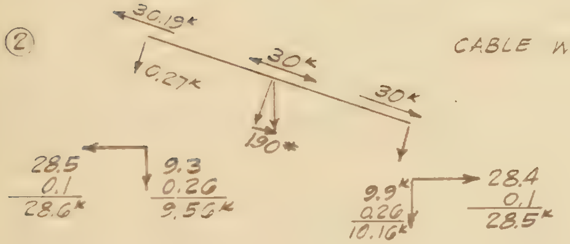
TYPICAL ARRANGEMENT
12 EQUAL ARCS

SHEET 33

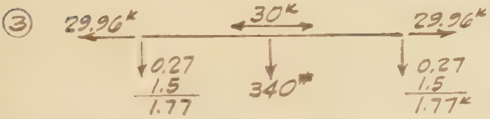
GUY CABLE ANALYSIS (CONTINUED)
FOCAL TOWER VERTICAL



CABLE WT = 1.8(356.733) = 640#

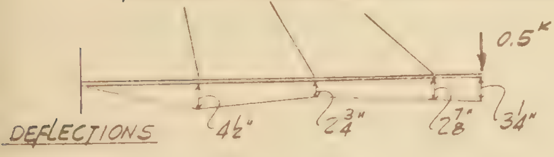


CABLE WT = 1.8(316.89) = 570#



CABLE WT = 1.8(300.37) = 540#

FOCAL TOWER HORIZONTAL



GUY CABLE ANALYSIS (CONTINUED)
EFFECT OF WIND

WIND LOADS:

30 MPH = $0.0033(\overline{30}^2)\frac{\pi}{4} = 2.33$ PSF

100 MPH = $0.0033(\overline{100}^2)\frac{\pi}{4} = 26.2$ PSF

AT 30 MPH FOR 24" OD PIPE

$w = 4.7 + 25\%$ (GUYS) $= 6$ #/FOOT

$D.L = \frac{6}{126} = 4.8\%$ OF HORIZ. CASE

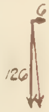
AT 100 MPH

$w = 52.4 + 25\% = 65.5$ #/FOOT

$D.L = \frac{65.5}{126} = 52\%$ OF HORIZ. CASE

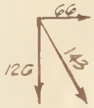
COMBINED ACTION - WIND PLUS DEAD LOAD

AT 30 MPH



0% INCREASE OVER HORIZ. CASE

AT 100. MPH



$\frac{17}{126} = 13\%$ INCREASE OVER HORIZ. CASE

∴ MAXIMUM DEFLECTION OF INSTRUMENT UNDER WIND PLUS DEAD LOAD

$= 3\frac{1}{4}" + 13\% = 3\frac{3}{4}"$ APPROX. USING GLASS FIBER CABLES

GUY CABLE ANALYSIS (CONTINUED)

FOCAL TOWER VERTICAL

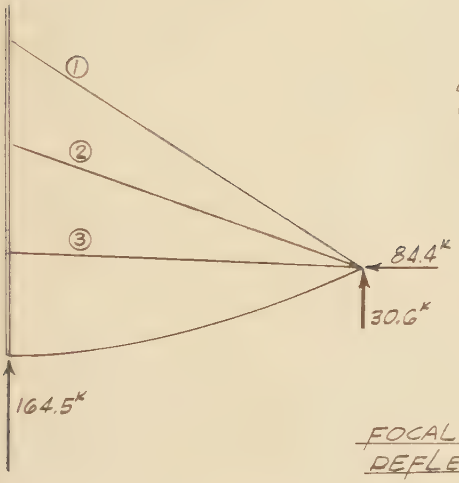
DEAD LOAD ONLY

INITIAL STRESS = 30^{K}

STEEL CABLES: $1\frac{11}{16}$" GALVANIZED PRESTRESSED CONCRETE WIRE STRAND

WEIGHT = 6#/FOOT

$AE = 1.73 \times 24 \times 10^{6} = 41.5 \times 10^{6}$



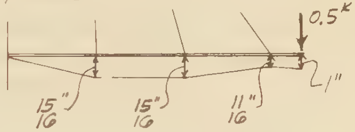
UNSTRETCHED LENGTHS

① 357.548'

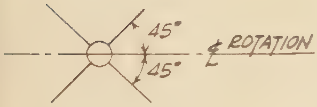
② 317.61'

③ 300.15'

FOCAL TOWER HORIZONTAL DEFLECTIONS

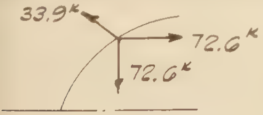


WITH WIND + DEAD LOAD DEFLECTION = $1\frac{1}{8}$"



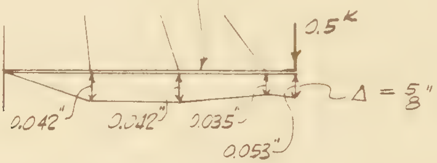
TYPICAL ARRANGEMENT ALL LEVELS

MAXIMUM LOADS ON RING GIRDER (3 CABLES GUYED AT ONE PANEL POINT)

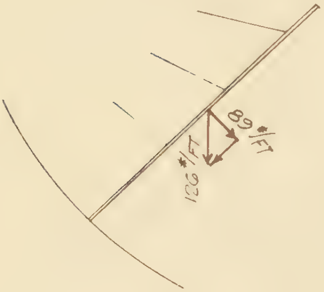


GUY CABLE ANALYSIS (CONTINUED)
STEEL CABLES
WIND IN PLANE OF ONE CABLE

FOR 100 MPH WIND

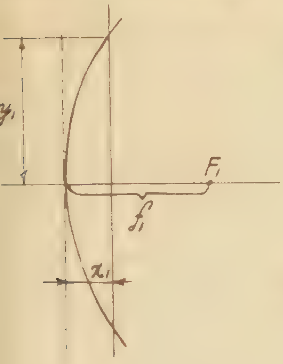


TOWER AT 45°
STEEL CABLES



FOR TOWER AT 45° DEFLECTIONS ARE $\frac{89}{126}$ = 70% OF HORIZONTAL CASE

ANTENNA UNDER TEMPERATURE EXPANSION



$$y_1^2 = 4f_1x_1 \qquad (1)$$

$$y_2 = y_1(1+\alpha)$$
$$x_2 = x_1(1+\alpha)$$

NEW PARABOLA

$$y^2 = 4f_2x_2$$

$$y_1^2(1+\alpha)^2 = 4f_2x_1(1+\alpha) \qquad (2)$$

DIVIDE (2) BY (1)

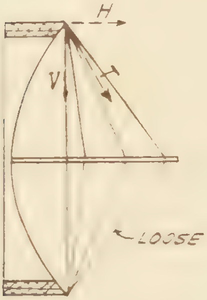
$$\frac{4f_2x_1\cancel{(1+\alpha)}}{4f_1x_1} = \frac{\cancel{y_1}^2(1+\alpha)^2}{\cancel{y_1}^2}$$

$$\frac{f_2}{f_1} = 1+\alpha$$

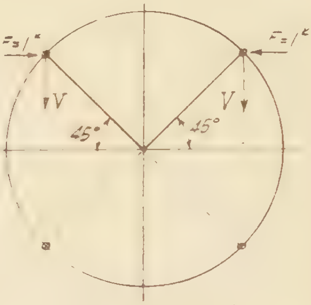
$$\underline{\underline{f_2 = f_1(1+\alpha)}}$$

TEMPERATURE CHANGES AFFECTING THE DIMENSIONS OF THE PARABOLOID WILL HAVE A COMPARABLE EFFECT ON THE FOCAL TOWER, THEREFORE THE PARABOLOID WILL BE IN FOCUS UNDER UNIFORM TEMPERATURE CHANGES.

EFFECT OF CABLES SUPPORTING ANTENNA ON RING



SECTION

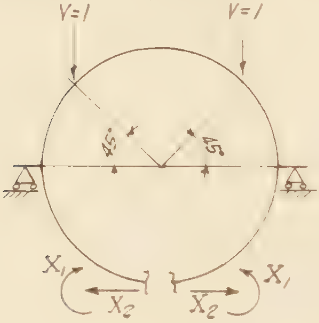


ELEVATION

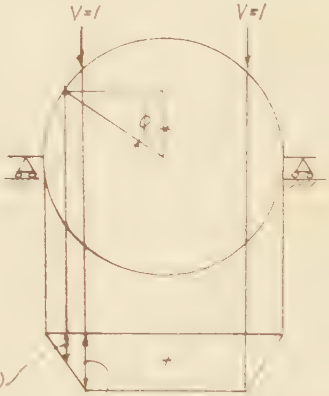
NOTE: H, V & F ARE COMPONENTS OF T

(1) RING LOADED V=1^k AT 45° APPLICATIONS OF GUY CABLE REACTION

SCHEME OF FORCES



M_o



$M_o = r(1 - \sin\phi)$

$M_o = r(1 - 0.707) = 0.293\,r$

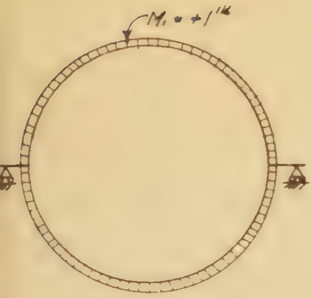
SIGN CONVENTION

+M = TENSION INSIDE
−M = TENSION OUTSIDE

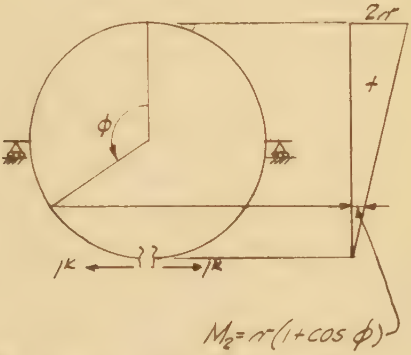
$M_o\ \phi = \pi/4$ TO $\phi = \pi/2$ $= r(1 - \sin\phi)$

$M_o\ \phi = 0$ TO $\phi = \pi/4$ $= 0.293\,r$ = CONSTA

$\underline{_1}$



$\underline{M_2}$



$$I\delta_{1m} = 2\int_0^{\pi/2} M_0 M_1 r\,d\phi = 2\int_0^{\pi/4} M_0 M_1 r\,d\phi + 2\int_{\pi/4}^{\pi/2} M_0 M_1 r\,d\phi =$$

$$= 2\int_{\pi/4}^{\pi/2} r(1-\sin\phi)\cdot \underset{=1}{M_1}\, r\,d\phi + 2\int_0^{\pi/4} 0.293r\cdot 1\cdot r\,d\phi = 0.623r^2$$

$$I\delta_{2m} = 2\int_0^{\pi/2} M_0 M_2 r\,d\phi = 2\int_0^{\pi/4} M_0 M_2 r\,d\phi + 2\int_{\pi/4}^{\pi/2} M_0 M_2 r\,d\phi = 1.123r^3$$

$$I\delta_{11} = \int_0^{2\pi} M_1^2 r\,d\phi = \int_0^{2\pi} 1^2\cdot r\,d\phi = r\int_0^{2\pi} d\phi = 2\pi r$$

$$I\delta_{22} = 2\int_0^{\pi} M_2^2 r\,d\phi = 2\int_0^{\pi} r^2(1+\cos\phi)^2 r\,d\phi = 9.45r^3$$

$$I\delta_{21} = EI\delta_{12} = \int_0^{\pi} \underset{=1}{M_1} M_2 r\,d\phi = 2\int_0^{\pi} r(1+\cos\phi) r\,d\phi = 6.28r^2$$

LASTIC EQUATIONS

$$X_1\delta_{11} + X_2\delta_{12} + \delta_{1m} = 0$$

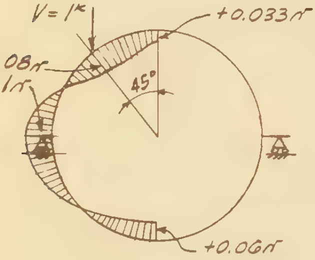
$$\underline{X_1\delta_{12} + X_2\delta_{22} + \delta_{2m} = 0}$$

$$(2\pi r)X_1 + (2\pi r^2)X_2 + 0.623r^2 = 0$$

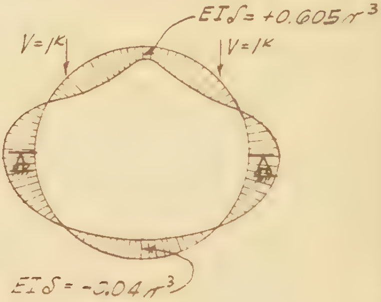
$$\underline{(2\pi r^2)X_1 + (3\pi r^3)X_2 + 1.123r^3 = 0}$$

$$X_1 = +0.06r$$
$$X_2 = -0.16$$

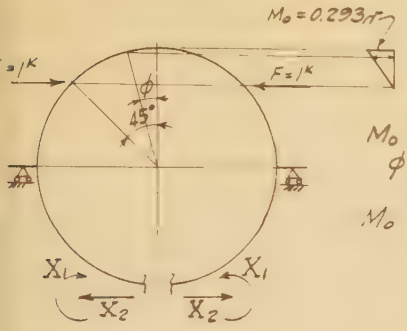
MOMENT DIAGRAM



VERTICAL DEFLECTIONS



ING UNDER $F = 1^K$ AT 45° APPLICATIONS OF GUY CABLE REACTION.



M_1 & M_2 DIAGRAMS SAME AS SHEET 12

$$M_o = -1\cdot r(\cos\phi - 0.707) = -r(\cos\phi - 0.707)$$
$$\phi = 0 \text{ TO } \pi/4$$

$$M_o = 0 \quad \text{FOR REST}$$

$$\left.\begin{aligned} EI\delta_{11} &= 2\pi r \\ EI\delta_{22} &= 3\pi r^3 \\ EI\delta_{12} &= 2\pi r^2 \end{aligned}\right\} \text{SEE SHEET 12}$$

$$EI\delta_{1m} = 2\int_0^{\pi/4} M_o M_1 r\,d\phi = -2\int_0^{\pi/4} 2(\cos\phi - 0.707)r\,d\phi = -0.295r^2$$

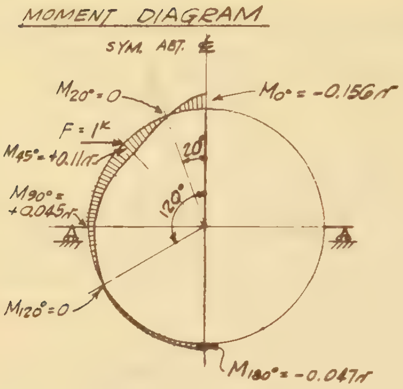
$$EI\delta_{2m} = 2\int_0^{\pi/4} M_o M_2 r\,d\phi = -2\int_0^{\pi/4} r(\cos\phi - 0.707)r(1+\cos\phi)r\,d\phi = -0.584r^3$$

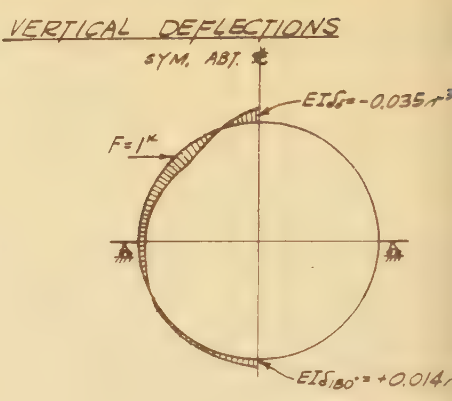
LASTIC EQUATIONS

$$X_1\delta_{11} + X_2\delta_{12} + \delta_{1m} = 0$$
$$X_1\delta_{12} + X_2\delta_{22} + \delta_{2m} = 0$$

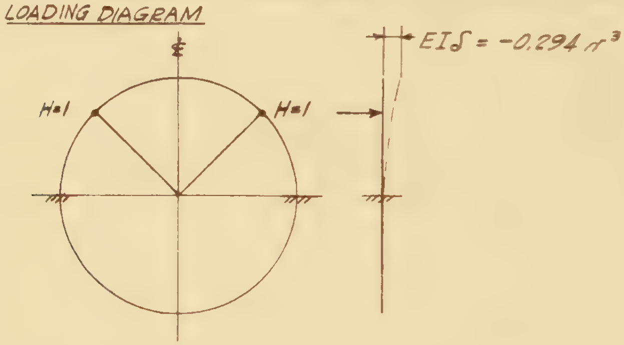
$$(2\pi r)X_1 + (2\pi r^2)X_2 - 0.295r^2 = 0$$
$$(2\pi r^2)X_1 + (3\pi r^3)X_2 - 0.584r^3 = 0$$

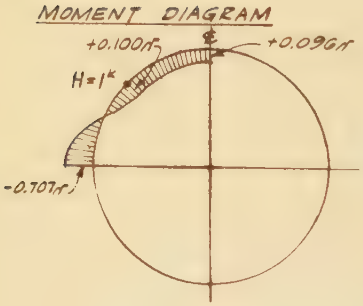
$$X_1 = -0.047r$$
$$X_2 = 0.092^K$$





(3) RING IN VERTICAL POSITION UNDER H = 1





DEFLECTION OF RING GIRDER DUE TO GUY CABLE REACTIONS

MAX REACTIONS TAKEN FOR RING IN VERTICAL POSITION

$V = F = 72.6^{K}$ USE AVERAGE VALUES OF $I_{x-x} = 3500\ ft^4$

$H = 33.9^{K}$

$I_{y-y} = 446\ ft^4$

$E = 4.16 \times 10^6\ K/ft^2$

$A = 415\ ft^2$

FOR V & F USE I_{y-y}

FOR H USE I_{x-x}

DEFLECTIONS FOR PARABOLOID WITH L=600' & H=75'

POINT	VERTICAL DEFLECTIONS (INCHS)		
	$V = 72.6^{K}$	$F = 72.6^{K}$	TOTAL
0°	+7.6	-0.5	+7.1
180°	-0.5	+0.2	-0.3

DEFLECTION DUE TO $H = 33.9^{K}$

$$\delta = \frac{0.294 r^3}{E I_{x-x}} = \frac{0.294 \times (300)^3 \times 12 \times 33.9}{4.16 \times 10^6 \times 3500} = -0.221 \text{ in}$$

THIS DEFLECTION WILL REDUCE DEFLECTIONS DUE TO DL & LL SHOWN ON SHEET

NOTE:

ASSUMPTION WAS MADE IN THE ABOVE CALCULATIONS THAT NO RESISTANCE TO HORIZONTAL MOVEMENT IS PROVIDED AT THE BEARINGS AT THE TOWERS. BUT THIS RESISTANCE EXISTS AND WILL DECREASE THE ABOVE DEFLECTIONS CONSIDERATELY.

ALSO THE PORTION OF THE ABOVE DEFLECTIONS DUE TO PRETENSIONING CAN BE TAKEN CARE OF BY PRE-CAMBERING OF THE RING GIRDER. THE ACTUAL DEFLECTIONS WILL BE DUE ONLY TO $\Delta T = 5^{K}$ (ΔT = MAX DIFFERENTIAL TENSION IN GUY CABLES DUE TO ROTATION OF DISC.)

STRESSES IN RING DUE TO GUY CABLE LOADS

$L = 600\ ft$

$H = 75\ ft$

$I_{Y-Y} = 446\ ft^4$ $I_{X-X} = 3500\ ft^4$

$c_{Y-Y} = 12.5\ ft$ $c_{X-X} = 47.0\ ft$

POINTS	MOMENT			$\Delta\sigma = \frac{Mc}{I}$	MOMENT DUE TO $H = 33.9$	$\Delta\sigma = \frac{Mc}{I}$
	DUE TO $F = 72.6$	DUE TO $V = 72.6$	TOTAL			
0°	$-0.156 \times 300 \times 72.6 = -3400^{\prime K}$	$+0.033 \times 300 \times 72.6 = +720^{\prime K}$	$-2680^{\prime K}$	520 psi	$+0.09 \times 300 \times 33.9 = 920$	85 psi
45°	$+0.11 \times 300 \times 72.6 = +2400^{\prime K}$	$+0.08 \times 300 \times 72.6 = +1740^{\prime K}$	$+4140^{\prime K}$	800	$+0.10 \times 300 \times 33.9 = 1020$	95 psi
90°	$+0.045 \times 300 \times 72.6 = +980^{\prime K}$	$-0.1 \times 300 \times 72.6 = -2170^{\prime K}$	-1190	230	$-0.707 \times 300 \times 33.9 = 7200$	675 psi
180°	$-0.047 \times 300 \times 72.6 = -1020^{\prime K}$	$+0.06 \times 300 \times 72.6 = +1300^{\prime K}$	$+280$	55		

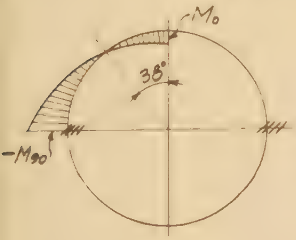
CONCLUSION: ADDITIONAL STRESSES DUE TO GUY CABLE LOADS
ARE SMALL COMPARED TO DL + LL STRESSES

TABULATION FOR RING IN HORIZONTAL POSITION
(ASSUMPTION RING CARRIES 0% LOAD OF SHELL)

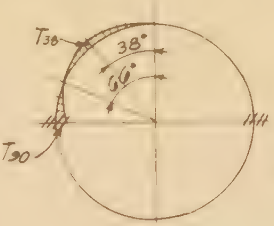
10 psf DL
10 psf LL
TOTAL LOAD = 20 psf

h ft	75'	[illegible]	[illegible]5'	[illegible]	50'
L ft	600'	600	600'	500	400'
A_K ft^2	302 000	[illegible] 3 000	327 000	[illegible]	33 500
I ft^4	3500	2625	1750	1400	1550
LOAD FROM SHELL KIPS	6040^{K}	$6[illegible]^{K}$	6540^{K}	4180^{K}	$[illegible]^{K}$
DEAD WEIGHT OF RING KIPS	3760^{K}	2320^{K}	1880^{K}	2570^{K}	1660^{K}
TOTAL LOAD KIPS	9800^{K}	9080^{K}	8420^{K}	6750^{K}	4330^{K}
p = 100% OF SHELL LOAD w	5.20 K/ft	4.80	4.55	4.35	3.42 K/ft
δ	2.2 in	15.0	21.0	7.0	3.56 in
p = 50% OF SHELL LOAD w	3.6 K/ft	3.4	2.75	2.97	2.37 K/ft
δ	8.4 in	10.0	12.7	4.8	2.46 in
p = 20% OF SHELL LOAD w	2.65 K/ft	2.5	1.69	2.17	.69 K/ft
δ	6.2 in	6.8	8.0	3.5	.76 in
p = 0% OF SHELL LOAD w	2.0 Kip/ft	1.50	1.0	1.64	.27 K/ft
δ	4.7 in	4.75	4.7	2.7	1.32 in
$M_{0°}$ FT. KIPS	126000^{IK}	123000^{IK}	126000^{IK}	73500	4[illegible]300
$-M_{90°}$ FT KIPS	-467000^{IK}	-455000^{K}	-467000^{IK}	-272000	-153000
$T_{38°}$ FT KIPS	-59000^{K}	-54500^{K}	-56000^{K}	-32600	-18300
$T_{90°}$ FT KIPS	140000^{IK}	136000^{K}	140000^{IK}	81500	46000
$V_{45°}$ KIPS	1220^{K}	1190^{K}	1230^{K}	850^{K}	600^{K}
$V_{90°}$ KIPS	2450^{K}	2400^{K}	2460^{K}	1700^{K}	1200^{K}

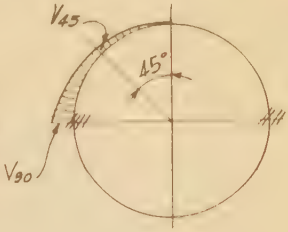
* VALUES SHOWN HERE ARE FOR 100% OF DL+LL ON RING GIRDER



M DIAGRAM



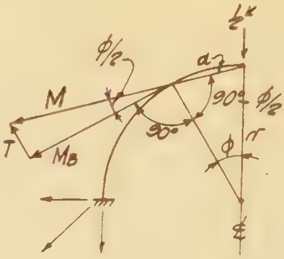
T DIAGRAM



V DIAGRAM

DEFLECTION DUE TO LOAD w K/ft

$$EI\delta = 2\int_0^{\pi/2} M M_1 r \, d\phi$$



$$M_1^{1K} = -\tfrac{1}{2}a = -\tfrac{1}{2} \times 2r \sin \phi/2 = 2 \sin \phi/2$$

$$M_1^{B} = M_1 \cos \phi/2 = -2 \sin \phi/2 \cos \phi/2 = -\tfrac{1}{2} r \sin \phi$$

$$EI\delta = -2\int_0^{\pi/2} w r^2 (U\cos\phi - 1)\tfrac{1}{2} r \sin\phi \, r \, d\phi$$

$$= \left(+\frac{2}{\pi} - 1\right) w r^4 = +0.36\, w r^4$$

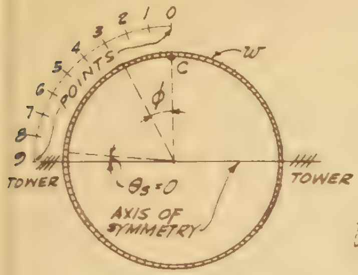
$$\delta_{℄} = \frac{+0.36\, w r^4}{EI}$$

RING IN VERTICAL POSITION

SHELL IS ASSUMED TO BE THE MAIN SUPPORTING MEMBER FOR THE VERTICAL LOAD. RING GIRDER IS DESIGNED TO RESIST THE STRESSES DUE TO GUY CABLES FROM THE FOCAL TOWER.

GENERAL EQUATIONS FOR RING IN HORIZONTAL POSITION

WITH UNIFORM LOAD w



$$M_c = wr^2(U-1)$$

$$M_{(BENDING)} = wr^2(U\cos\phi - 1)$$

$$T_{(TORSION)} = -wr^2(U\sin\phi - \phi)$$

$$U = \frac{2(q+1)\cos\theta_s - 2q\left(\frac{\pi}{2}-\theta_s\right)\sin\theta_s}{(q+1)\left(\frac{\pi}{2}-\theta_s\right) - (q-1)\sin\theta_s\cos\theta_s}$$

$$\theta_s = 0 \begin{cases} \sin\theta_s = 0 \\ \cos\theta_s = 1 \end{cases}$$

I.E. θ_s IS THE ANGLE BETWEEN THE AXIS OF SYMMETRY AND THE SUPPORT

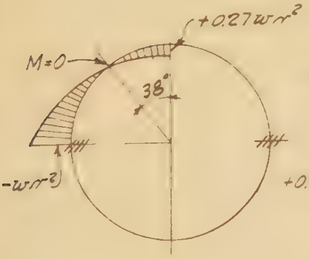
WHERE $q = \frac{EI}{GK}$

$$U = \frac{2(q+1)}{\pi/2\,(q+1)} = \frac{4}{\pi} = 1.27$$

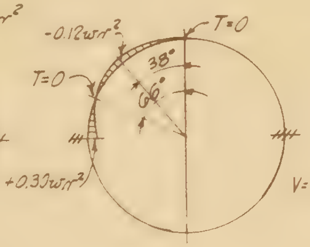
$$M_c = wr^2\left(\frac{4}{\pi}-1\right) = wr^2(1.27-1) = 0.27\,wr^2$$

$$M = wr^2\left(\frac{4}{\pi}\cos\phi - 1\right) = wr^2(1.27\cos\phi - 1)$$

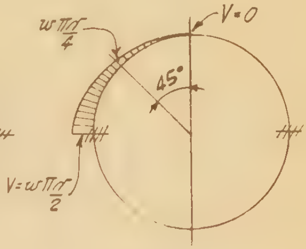
$$T = -wr^2\left(\frac{4}{\pi}\sin\phi - \phi\right) = -wr^2(1.27\sin\phi - \phi)$$



M DIAGRAM



T DIAGRAM



V DIAGRAM

DESIGN OF RING (IN HORIZONTAL POSITION)

SHELL LOAD: D.L. = 10 PSF
L.L. = 10
TOTAL = 20 PSF

PARABOLOID: L = 600'
H = 75'
AREA = 302 000 ft^2
W @ 20 PSF = 6040^K

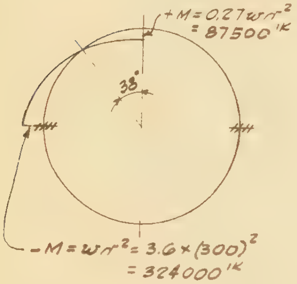
FOR CALCULATION OF RING TAKE 50% OF SHELL LOAD + RING D.L.
50% OF SHELL LOAD = 6040 × 0.50 = 3020^K

$w_{SHELL} = \frac{3020}{2\pi r} = \frac{3020}{2 \times 3.14 \times 300} = 1.60\ ^K/ft$

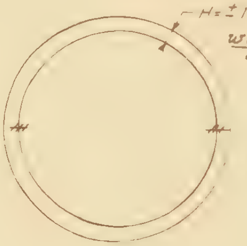
ASSUME: $W_{RING} = 2.0$

$W_{TOTAL} = 3.6\ ^K/ft$ (UNIFORM DISTRIBUTION)

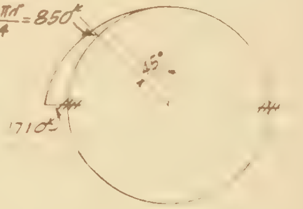
TOTAL H = CONSTANT ALONG THE RING = $6.4 \times \frac{600}{2} = \pm 1930^K$



M DIAGRAM

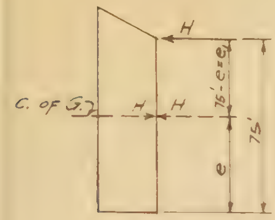


H DIAGRAM



V DIAGRAM

EFFECT OF ECCENTRICITY OF HORIZONTAL RING FORCE



DUE TO H ACTING AT ECCENTRICITY e, ADDITIONAL POSITIVE MOMENTS IN THE RING ARE OBTAINED.

ALLOWABLE STRESSES FOR BUCKLING

USE HIGH STRENGTH STEEL (YIELD POINT = 50 000 psi)

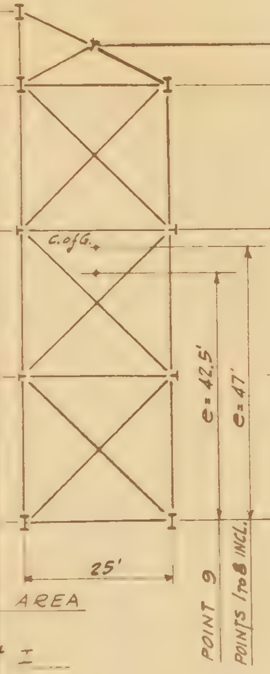
MEMBER	r_{MIN}	L ft	L/r	ALLOWABLE STRESS (PSI)
14 WF 119	3.75	25.13	81	18730
14 WF 287	4.17	25.13	72	9520
14 WF 398	4.31	25.13	70	19690
ST 8WF29	1.88	25.13	160	9360
ST 13 WF 57	2.11	25.13	142	11250

MOMENTS AND SHEAR IN RING GIRDER

POINT	ϕ	SIN ϕ	COS ϕ	M_B (1K)	$Me = H \times e$, (1K)	$M_B + Me$	AREA (in^2)	I (ft^4)	f_N (psi)	f_M (psi)	$f_N + f_M$ (psi)	V (K)	LOAD (K) IN DIAG.	DIAGONAL MEMBERS
0	0	0.000	1.000	+87500	+54000	+141500	505	3861	3800	11900	15700	0	0	5"ϕ 14.62
1	10	0.174	0.985	+81000	+54000	+135000	505	3861	3800	11400	15200	190	19.0	5"ϕ 14.62
2	20	0.342	0.940	+64800	+54000	+118800	505	3861	3800	10000	13800	380	38.0	5"ϕ 14.62
3	30	0.500	0.866	+32400	+54000	+86400	505	3861	3800	7300	11100	570	57.0	8"ϕ 24.7
4	40	0.643	0.766	− 9700	+54000	+44300	218	1968	8900	7400	16300	760	76.0	8"ϕ 24.7
5	50	0.766	0.643	−58000	+54000	− 4000	218	1968	8900	800	9700	950	95.0	8"ϕ 24.7
6	60	0.866	0.500	−118000	+63000	−55000	218	1968	8900	10600	19500	1140	114.0	10"ϕ 34.24
7	70	0.940	0.342	−182000	+54000	−128000	505	3861	3800	10900	14700	1350	135.0	10"ϕ 34.24
8	80	0.985	0.174	−250000	+54000	−196000	505	3861	3800	16600	20400	1520	152.0	10"ϕ 34.24
9	90	1.000	0.000	−324000	+63000	−261000	604	5848	3200	14000	17200	1710	171.0	PLATES

SECTIONS OF RING GIRDER FOR H=75' & L=600

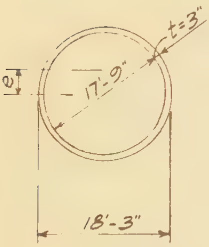
POINTS 0, 1, 2, 3 7 & 8	POINTS 4, 5 & 6	POINT 9	
14 WF 287 84.37□"	14 WF 103 30.26□"	14 WF 314 92.30□"	
14 WF 287 84.37□"	14 WF 119 34.99□"	14 WF 314 92.30□"	
ST 13 WF 57 16.77□"	ST 8 WF 29 8.52□"	ST 13 WF 57 16.77□"	
ST 13 WF 57 16.77□"	ST 8 WF 29 8.52□"	ST 13 WF 57 16.77□"	
14 WF 287 84.37□"	14 WF 119 34.99□"	14 WF 398 116.98□"	
505□"	218□"	604□"	AREA
3861 ft^4	968 ft^4	5848 ft^4	I



POINTS 0, 1, 2, 3 7 & 8	POINTS 4, 5 & 6	POINT 9
ST 13 WF 57 16.77□"	ST 8 WF 29 8.52□"	ST 13 WF 57 16.77□"
14 WF 287 84.37□"	14 WF 119 34.99□"	14 WF 314 92.30□"
ST 13 WF 57 16.77□"	ST 8 WF 29 8.52□"	ST 13 WF 57 16.77□"
ST 13 WF 57 16.77□"	ST 8 WF 29 8.52□"	ST 13 WF 57 16.77□"
14 WF 287 84.37□"	14 WF 119 34.99□"	14 WF 398 116.98□"

DESIGN OF MAIN CONNECTING SHAFT

1) INNER SHAFT



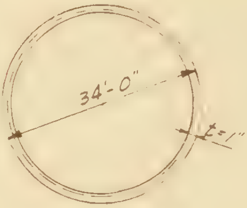
$$A = \frac{\pi}{4}\left[(18.25)^2 - (17.75)^2\right] = 14.2\ ft^2$$

$$I = 3.14 \times 0.25 \times \left(\frac{18.0}{2}\right)^3 = 578\ ft^4$$

$$e = 0.637 \times 9.0 = 5.72'$$

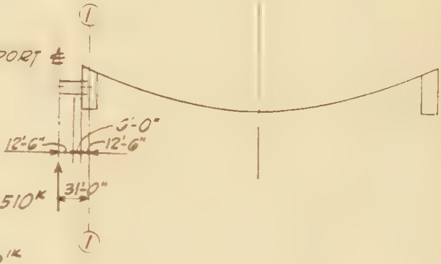
$$Q = \frac{A}{2} \times e = 14.2 \times \frac{1}{2} \times 5.72 = 40.6\ ft^3$$

2) OUTER SHAFT



$$A = 0.788 \times (33.96)^2 = 901\ ft^2$$

(a) DL + 10 w



$$\begin{aligned} &\text{TOTAL D.L.} &&= 10\,000^K \\ &302000\ ft^2 @ 10w &&= \underline{\ \ 3020\ \ } \\ & && \quad 13020^K \end{aligned}$$

$V_{D.L.+L.L.}$ AT EA. TOWER $= \frac{1}{2} \times 13020 =$ $V = 6510^K$

$M_{\text{BENDING AT 1-1}} = 6510^K \times 31' = 202000'^K$

$$S_v = \frac{V \times Q}{I \times b} = \frac{6510^K \times 40.6}{578 \times 0.5 \times 144} = 6.4\ ksi = 6400\ psi$$

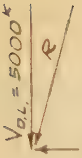
$$S_f = \frac{Mc}{I} = \frac{202\,000'^K \times 9.1}{578 \times 144} = 22.1\ ksi = 22100\ psi$$

PRINCIPAL STRESSES

$$\text{MAX SHEAR} = \sqrt{(6.4)^2 + \left(\frac{22.1}{2}\right)^2} = 12.8\ ksi = 12800\ psi < 13000\ psi$$

$$\text{NORMAL STRESS} = \frac{22.1}{2} + 12.8 = 23.9\ ksi = 23900\ psi < 27270\ psi$$

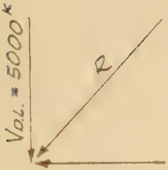
(b) D.L. + 10* w



$\frac{3020}{2} = 1510^k = W_{10}$

$R = \sqrt{(5000)^2 + (1510)^2} = 5200^k < 6510^k$ USED IN a) LOAD

(c) D.L. + 30* w



$W_{30} = 1510 \times \frac{30}{10} = 4530^k$

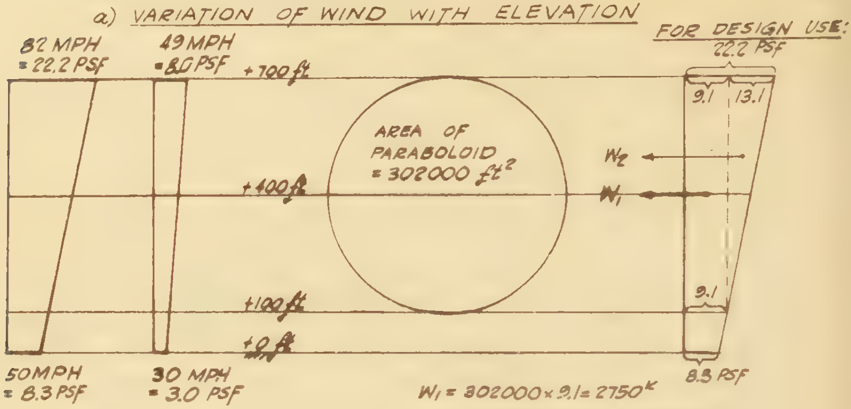
$R = \sqrt{(5000)^2 + (4530)^2} = 6720^k$

$\frac{6720}{6510} = 1.04 < 1.33$ ALLOWABLE INCREASED STRESS FOR COMBINED D.L. + W.L.

(d) DEAD LOAD WITH WIND VARYING IN HEIGHT

1) SHELL IN VERTICAL POSITION

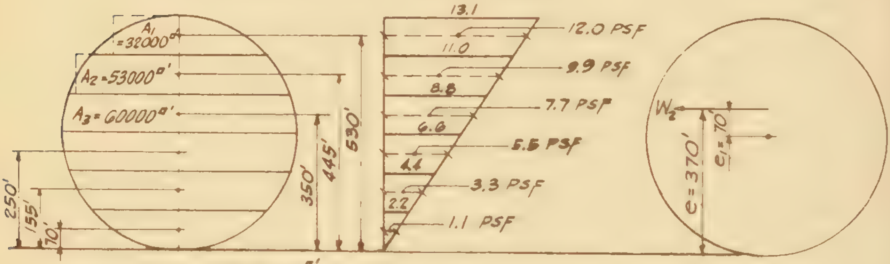
a) VARIATION OF WIND WITH ELEVATION



$W_1 = 302000 \times 9.1 = 2750^K$

$W_2 = 302000 \times 13.1 \times \frac{1}{2} = 1980^K$

b) CENTER OF GRAVITY OF W₂



TOTAL AREA = 290 000□'

$$W_2\,e = \Big[\overset{204000^K}{32000 \times 12.0 \times 580} + \overset{231000^K}{53000 \times 9.9 \times 445} + \overset{162000^K}{60000 \times 7.7 \times 350} + \overset{82500^K}{60000 \times 5.5 \times 250}$$

$$+ \underset{27000^K}{53000 \times 3.3 \times 155} + \underset{2500^K}{32000 \times 1.1 \times 70}\Big]$$

$$= 709000^K$$

$$e = \frac{709\,000^K}{1980} = 355'$$

CORRECT VALUE FOR PARABOLOID $= \frac{302\,000}{290\,000} = 1.04$ $\quad e = 355 \times 1.04 = 370$ ft

M_T = TORQUE AT EACH TOWER $= \frac{1}{2} W_2 \times e_1 = \frac{1}{2} \times 1980 \times 70 = 69000^{'K}$

TORQUE EA. TOWER = $69000^{\prime K}$

TRY: $D = 34'-0"$

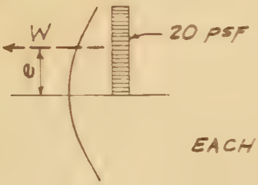
$d = 33'-11"$

$t = 1"$

$A = 0.788 \times (33.96)^2 = 901\ ft^2$

$$S_T = \frac{M_T}{2At} = \frac{69000}{2 \times 901 \times 0.083 \times 144} = 3200\ psi \quad \text{O.K.}$$

ASSUME:



$e = 0.424R = 0.424 \times 300 = 127'$

$W = 20 \times 302\,000 \times \frac{1}{2} = 3020^K$

$M_T = 3020 \times 127 = 384000^{\prime K}$

EACH SIDE : $M_T = \frac{1}{2} \times 384000 = 192000^{\prime K}$

$$S_T = \frac{192\,000}{2 \times 901 \times 0.083 \times 144} = 8900\ psi < 13000\ psi$$

CONCLUSION :

SHAFT WILL BE ABLE TO RESIST 20 PSF DIFFERENTIAL WIND LOAD ON $\frac{1}{2}$ THE SHELL.

TOWER DESIGN

L = 600' & H = 75' SHELL

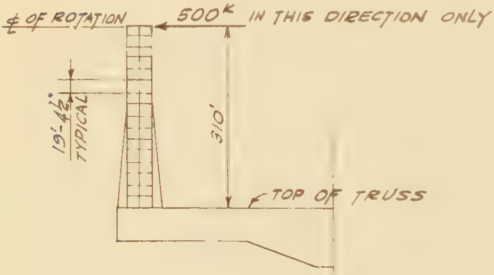
DESIGN LOADS:

(A) DEAD LOAD

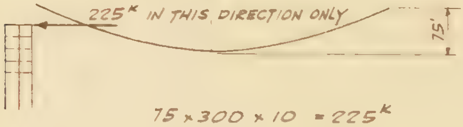
SHELL D.L. = 10.0 × 302 000 ft^2	=	3020^K
RING D.L = 2.0 × 600 × 3.14	=	3760
SHAFT WT. = 350 × 2	=	700
2 BRACING TRUSSES	=	920
OBSERVATION ROOM	=	34
38 CABLES & FITTINGS	=	61
PASSAGE WAY	=	84
MOTOR & GEARS	≈	900
FOCAL TOWER & CABLES	=	60
		9539^K PER 2 TOWER

DESIGN LOAD FOR ONE TOWER = 5000^K

(B) HORIZONTAL THRUST = 500^K

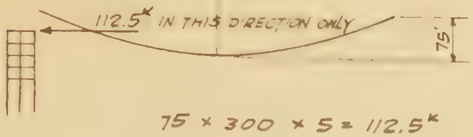


(C) 10 PSF WIND THRUST



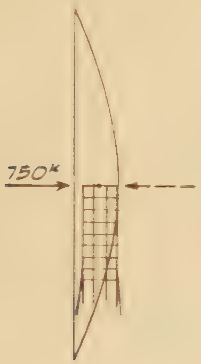
75 × 300 × 10 = 225^K

(D) 5 PSF WIND THRUST



$75 \times 300 \times 5 = 112.5^{K}$

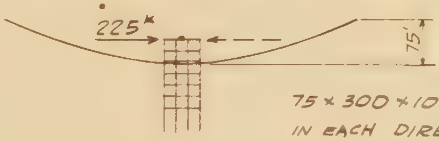
(E) 5 PSF WIND THRUST



$300\,000 \times 5 = 1500^{K}$ PER 2 TOWERS

750^{K} THRUST PER TOWER EACH DIRECTION

(F) 10 PSF WIND THRUST



$75 \times 300 \times 10 = 225^{K}$
IN EACH DIRECTION

MATERIAL:

HIGH STRENGTH STEEL (A242)
(50000 psi YIELD POINT)

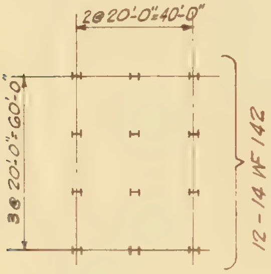
WORKING STRESSES:

NORMAL ALLOWABLE $\begin{cases} \text{TENSION} = 27270 \text{ psi} \\ \text{COMPRESSION} = \left[22500 - 0.738\left(\frac{\ell}{r}\right)^2\right] \text{psi} \end{cases}$

1) LOAD COMBINATION (A) + (B) + (C) DESIGN FOR 1.33 × NORMAL STRESSES
2) LOAD COMBINATION (A) + (B) + (D) DESIGN FOR NORMAL STRESSES
3) LOAD COMBINATIONS (A) + (B) + (E) DESIGN FOR FOR NORMAL STRESSES
4) LOAD COMBINATION (A) + (B) + (F) DESIGN FOR 1.33 × NORMAL STRESSES

∴ THE CRITICAL LOAD COMBINATION WILL BE 3): (A) + (B) + (E)

BASIC SECTION:



$A_1 = 41.85 \text{ in}^2$

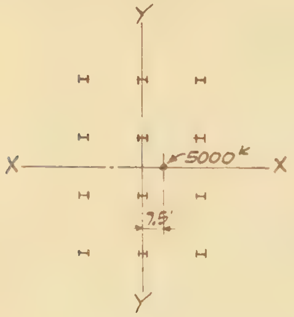
$r_{MIN} = 3.97 \text{ in}$

UNBRACED LENGTH = 19'-4½"

$\frac{\ell}{r} = \frac{19.40 \times 12}{3.97} = 58.6 - F_a = 20.53 \text{ ksi}$

$\frac{\ell \times d}{b \times t} = \frac{19.40 \times 12 \times 14.75}{15.50 \times 1.063} < 600$ o

TOP SECTION:



SECTION PROPERTIES:

$S_{y-y} = 8 \times 41.85 \times 20^{\circ} \times 12 = 80\,400 \text{ in}^3$

$I_{x-x} = 6 \times 41.85 (10 \times 12)^2 = 35.1 \times 10^5 \text{ in}^4$

$+ 6 \times 41.85 (30 \times 12)^2 = \underline{324.9 \times 10^5}$

$360.0 \times 10^5 \text{ in}^4$

$S_{x-x} = \frac{360 \times 10^5}{30 \times 12} = 100\,000 \text{ in}^3$

$f_a = \frac{5000}{12 \times 41.85} = 9.95 \text{ ksi}$

$M_{y-y} = 5000 \times 7.5 = 37400^{\text{lK}}$

$f_{b_{y-y}} = \frac{37400 \times 12}{80400} = 5.60 \text{ ksi}$

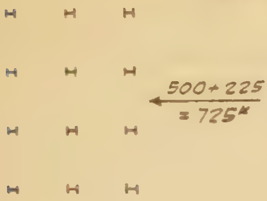
$f_a + f_{b_{y-y}} = 15.55 \text{ ksi}$

SHEET 59

ECTION A–A (50'-0" FROM TOP ℄ ROTATION):

(SECTION PROPERTIES SAME AS ON PREVIOUS SHEET)

(A) + (B) + (C) LOADING.



$M_{Y-Y} = 725 \times 50 - 5000 \times 7.5$

OR 5000×7.5 ONLY (= MAX)

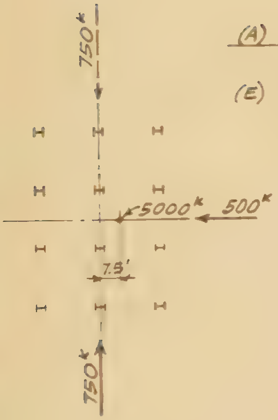
$= 37500^{\prime K}$

$f_{b_{Y-Y}} = \dfrac{37500 \times 12}{80\,400} = 5.60$ ksi

$f_a = \dfrac{5000 + 200 \text{(TOWER LOAD)}}{12 \times 41.85} = 10.40$ ksi

$f_a + f_{b_{Y-Y}} = 16.0$ ksi

(A) + (B) + (E) LOADING:



(E) LOADING $= 300\,000 \times 5 = 1500^K$ PER 2 TOWERS

$\therefore W = 750^K$ PER TOWER

$M_{Y-Y} = 500 \times 50 - 37400$

OR 5000×7.5 ONLY (= MAX)

$= 37500^{\prime K}$

$M_{X-X} = 750 \times 50 = 37500^{\prime K}$

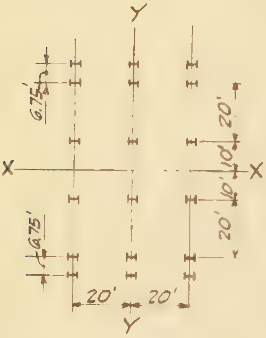
$f_{b_{Y-Y}} = 5.60$ ksi (AS ABOVE)

$f_{b_{X-X}} = \dfrac{37500 \times 12}{100\,000} = 4.50$ ksi

$f_a = 10.40$ ksi

$f_a + f_{b_{X-X}} + f_{b_{Y-Y}} = 20.5$ ksi

SECTION B-B (126'-0" FROM TOP ℄ ROTATION)



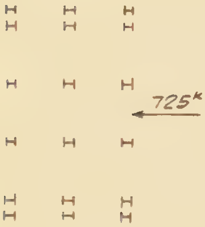
SECTION PROPERTIES:

$S_{Y-Y} = 12 \times 41.85 \times 20 \times 12 = 120\,000 \text{ in}^3$

$I_{X-X} = 360 \times 10^5 + 6 \times 41.85 (36.75 \times 12)^2 = 834 \times 10$

$S_{X-X} = \dfrac{834 \times 10^5}{36.75 \times 12} = 184\,000 \text{ in}^3$

(A) + (B) + (C) LOADING:



$M_{Y-Y} = 725 \times 126 - 37400 = 54000^{\prime K}$

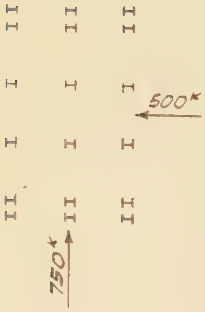
$f_{b\,Y-Y} = \dfrac{54000 \times 2}{120\,000} = 5.40 \text{ Ksi}$

$f_a = \dfrac{5000 + 700}{18 \times 41.85} = 7.70$

13.10 Ksi

∴ (A) + (B) + (D) LOADING O.K.

(A) + (B) + (E) LOADING:



$M_{Y-Y} = 500 \times 126 - 37400 = 25600^{\prime K}$

$\text{MAX } M_{Y-Y} = 37400^{\prime K}$

$M_{X-X} = 750 \times 126 = 94\,500^{\prime K}$

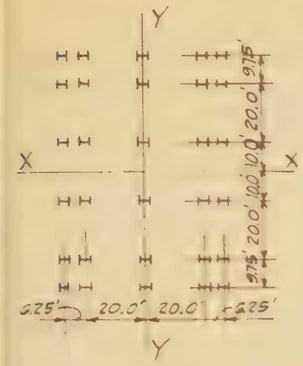
$f_{b\,Y-Y} = \dfrac{37400 \times 12}{120\,000} = 3.74 \text{ Ksi}$

$f_{b\,X-X} = \dfrac{94\,500 \times 12}{189\,000} = 6.00$

$f_a = 7.70$

17.44 Ksi

SECTION C-C (65'-0" FROM TOP ℄ OF ROTATION)



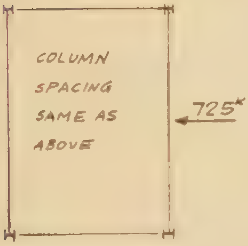
$$I_{Y-Y} = 12 \times 41.85(20 \times 12)^2 = 289 \times 10^5$$
$$+12 \times 41.85(26.25 \times 12)^2 = \underline{498 \times 10^5}$$
$$787 \times 10^5$$

$$S_{Y-Y} = \frac{787 \times 10^5}{26.25 \times 12} = 260\,000 \text{ in}^3$$

$$I_{X-X} = 10 \times 41.85(10 \times 12)^2 = 60 \times 10^5$$
$$10 \times 41.85(30 \times 12)^2 = 540 \times 10^5$$
$$10 \times 41.85(39.75 \times 12)^2 = \underline{950 \times 10^5}$$
$$1550 \times 10^5$$

$$S_{X-X} = \frac{1550 \times 10^5}{39.75 \times 12} = 325\,000 \text{ in}^3$$

(A) + (B) + (C) LOADING:



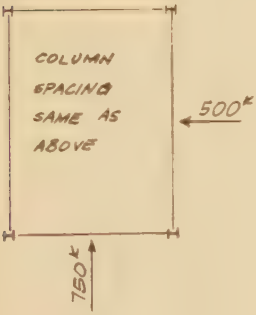
$$M_{Y-Y} = 725 \times 165 - 37400 = 81600^{\prime K}$$

$$f_{b\,Y-Y} = \frac{81600 \times 12}{260\,000} = 3.78 \text{ ksi}$$

$$f_a = \frac{(5000 + 900)}{30 \times 41.85} = \underline{4.70}$$

$$f_a + f_{b\,Y-Y} = 8.48 \text{ ksi}$$

(A) + (B) + (E) LOADING:



$$M_{Y-Y} = 500 \times 165 - 37400 = 45100^{\prime K}$$

$$M_{X-X} = 750 \times 165 = 123\,800^{\prime K}$$

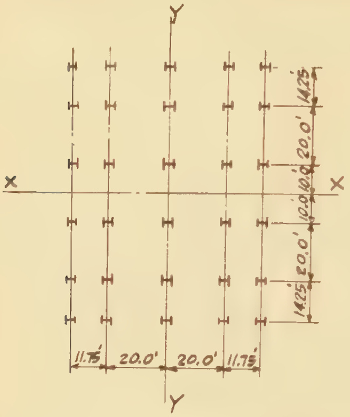
$$f_{b\,Y-Y} = \frac{45100 \times 12}{260\,000} = 2.08 \text{ ksi}$$

$$f_{b\,X-X} = \frac{123\,800 \times 12}{325\,000} = 4.56$$

$$f_a = \underline{4.70}$$

$$f_a + f_{b\,Y-Y} + f_{b\,X-X} = 11.34 \text{ Ksi}$$

SECTION D-D (223'-0" FROM TOP ℄ ROTATION)



SECTION PROPERTIES:

$$I_{Y-Y} = 12 \times 41.85(20.0 \times 12)^2 = 289 \times 10$$

$$+12 \times 41.85(31.75 \times 12)^2 \quad 728 \times 10$$

$$1017 \times 10^5 \text{ IN}$$

$$S_{Y-Y} = \frac{1017 \times 10^5}{31.75 \times 12} = 267\,500 \text{ IN}^3$$

$$I_{X-X} = \quad 60 \times 10^5$$

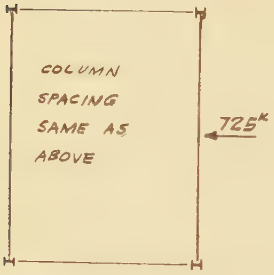
$$540 \times 10^5$$

$$+10 \times 41.85(44.25 \times 12)^2 = 1180 \times 10^5$$

$$1780 \times 10^5$$

$$S_{X-X} = \frac{1780 \times 10^5}{44.25 \times 12} = 335\,000 \text{ IN}^3$$

(A) + (B) + (C) LOADING:



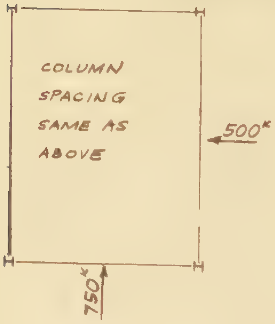
$$M_{Y-Y} = 725 \times 223 - 37400 = 124\,100^{\text{IK}}$$

$$f_{b\,Y-Y} = \frac{124\,100 \times 12}{267\,500} = 5.58^{\text{ksi}}$$

$$f_a = \frac{(5000 + 1200)}{30 \times 41.85} = 4.95$$

$$f_a + f_{b\,Y-Y} \quad 10.53^{\text{ksi}}$$

(A) + (B) + (E) LOADING:



$$M_{Y-Y} = 500 \times 223 - 37400 = 74\,100^{\text{IK}}$$

$$M_{X-X} = 750 \times 223 = 167\,200^{\text{IK}}$$

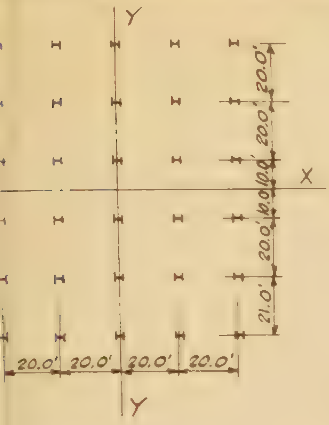
$$f_{b\,Y-Y} = \frac{74\,100 \times 12}{267\,500} = 3.33^{\text{ksi}}$$

$$f_{b\,X-X} = \frac{167\,200 \times 12}{335\,000} = 6.00$$

$$f_a = 4.95$$

$$f_a + f_{b_{Y-Y}} + f_{b_{X-X}} = 14.28^{\text{ksi}}$$

CTION E-E (310'-0" FROM TOP ℄ OF ROTATION i.e. ABOVE CANTILEVER TRUSS)



SECTION PROPERTIES:

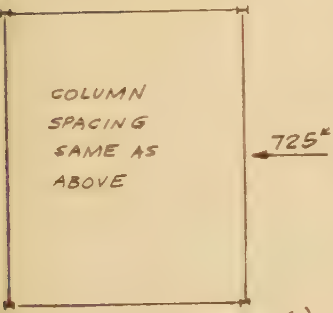
$$\begin{aligned} I_{Y-Y} &= & &= 289 \times 10^5 \\ &\quad +12 \times 41.85(40 \times 12)^2 & &= \underline{1156 \times 10^5} \\ & & &\quad 1445 \times 10^5 \text{ IN}^4 \end{aligned}$$

$$S_{Y-Y} = \frac{1445 \times 10^5}{40 \times 12} = 300\,000 \text{ IN}^3$$

$$\begin{aligned} I_{X-X} &= & &= 60 \times 10^5 \\ & & &\quad 540 \times 10^5 \\ &\quad +10 \times 41.85(51 \times 12)^2 & &= \underline{1565 \times 10^5} \\ & & &\quad 2165 \times 10^5 \end{aligned}$$

$$S_{X-X} = \frac{2165 \times 10^5}{51 \times 12} = 354\,000 \text{ IN}^3$$

(A) + (B) + (C) LOADING:

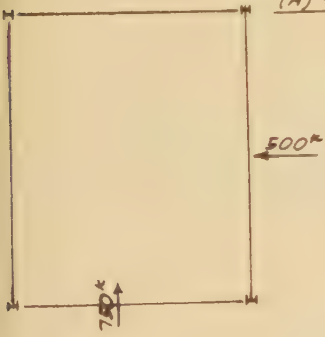


$$M_{Y-Y} = 725 \times 310 - 37400 = 187\,100^{'K}$$

$$f_{bY-Y} = \frac{187\,100 \times 12}{300\,000} = 7.50^{\text{ KSI}}$$

$$f_a = \frac{(5000 + 2000)}{30 \times 41.85} = 5.58$$

(A) + (B) + (E) LOADING:



$$M_{Y-Y} = 500 \times 310 - 37400 = 117600^{'K}$$

$$M_{X-X} = 750 \times 310 = 232\,000^{'K}$$

$$f_{bY-Y} = \frac{117600 \times 12}{300\,000} = 4.70^{\text{ KSI}}$$

$$f_{bX-X} = \frac{232\,000 \times 12}{354\,000} = 7.86$$

$$\begin{aligned} f_a & \qquad = \underline{5.58} \\ & \qquad\quad 18.14^{\text{ KSI}} \end{aligned}$$

TAKING INTO ACCOUNT WIND ACTION ON THE TOWER:

(A) + (B) + (C) LOADING

EXPOSED AREA:

$$6 \times \frac{15.5}{12} \times 310 = 2400^{\square'}$$

ADDITIONAL HORIZONTHAL THRUST OF $2400 \times 1.5 \times 10$ PSF $= 36^{K}$ AT MID-HEIGHT

$$M'_{Y-Y} = 36 \times \frac{310}{2} = 5590^{\prime K}$$

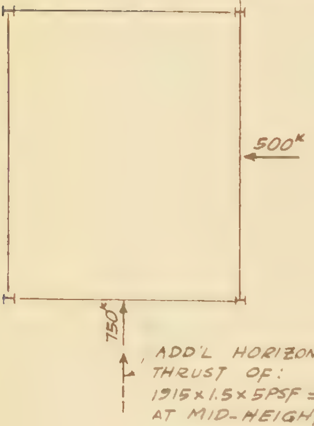
$$M_{Y-Y} + M'_{Y-Y} = 187\,100 + 5590 = 192\,690^{\prime K}$$

$$f_{b_{Y-Y}} = \frac{192\,690 \times 12}{300\,000} = 7.80$$

$$f_a = \underline{5.58}$$

$$13.38 \text{ ksi}$$

(A) + (B) + (E) LOADING



EXPOSED AREA:

$$5 \times \frac{14.75}{12} \times 310 = 1915^{\square'}$$

$$M'_{X-X} = 14.35 \times \frac{310}{2} = 2225^{\prime K}$$

$$M_{X-X} + M'_{X-X} = 232\,000 + 2225 = 234\,225^{\prime K}$$

$$f_{b_{X-X}} = \frac{234\,225 \times 12}{354\,000} = 7.95 \text{ ksi}$$

ADD'L HORIZONTAL THRUST OF:
$1915 \times 1.5 \times 5\text{PSF} = 14.35^{K}$
AT MID-HEIGHT

∴ MAXIMUM TOTAL STRESS:

$$f_a + f_{b_{Y-Y}} + f_{b_{X-X}} = 5.58 + 4.70 + 7.95 = 18.23 \text{ ksi}$$

WIND BASE OF TOWER

DESIGNED FOR $5^{\#}/\phi$ WIND THRUST IN EACH DIRECTION $(= 750^{K})$

$M = 750 \times 370 = 277000^{\prime K}$ FOR 3 TRUSSES IN EACH SIDE.

M PER TRUSS ON EACH SIDE $= \frac{277000}{3} = 92333^{K}$.

ASSUME $1^{K}/\text{FT.}$ D.L. FOR EACH TRUSS.

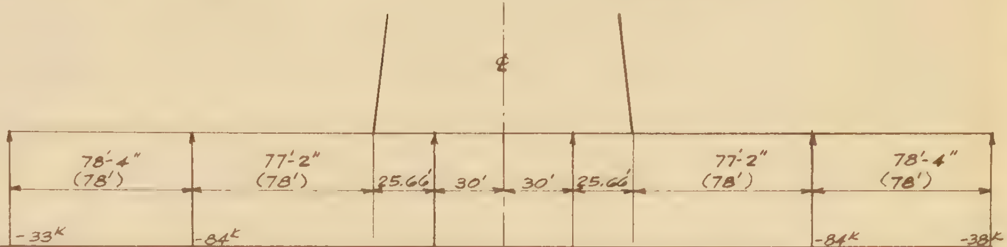
OUTSIDE TRUSS

$$I.A = 2\left[211.66^2 + 133.66^2 + 30^2\right] = 126700A$$

$$N_1 = \frac{92333 \times 211.66}{126700} = 154^{K}$$

$$N_2 = \frac{92333 \times 133.66}{126700} = 98^{K}$$

$$N_3 = \frac{92333 \times 30}{126700} = 22^{K}$$



TRUSS D.L.	-33^{K}	-84^{K}			-84^{K}	-38^{K}
MOMENT REACTION	-154^{K}	-98^{K}	(-22^{K})	$(+22^{K})$	$+98^{K}$	$+154^{K}$
SUMMATION	-187^{K}	-182^{K}			$+14^{K}$	$+121^{K}$
ADD'L. LOAD NEEDED	-121^{K}	-14^{K}			-14^{K}	-121^{K}
FINAL WHEEL LOAD (FOR ONE WIND DIRECTION)	-308^{K}	-196^{K}			0	0

SHEET 66

MIDDLE TRUSS

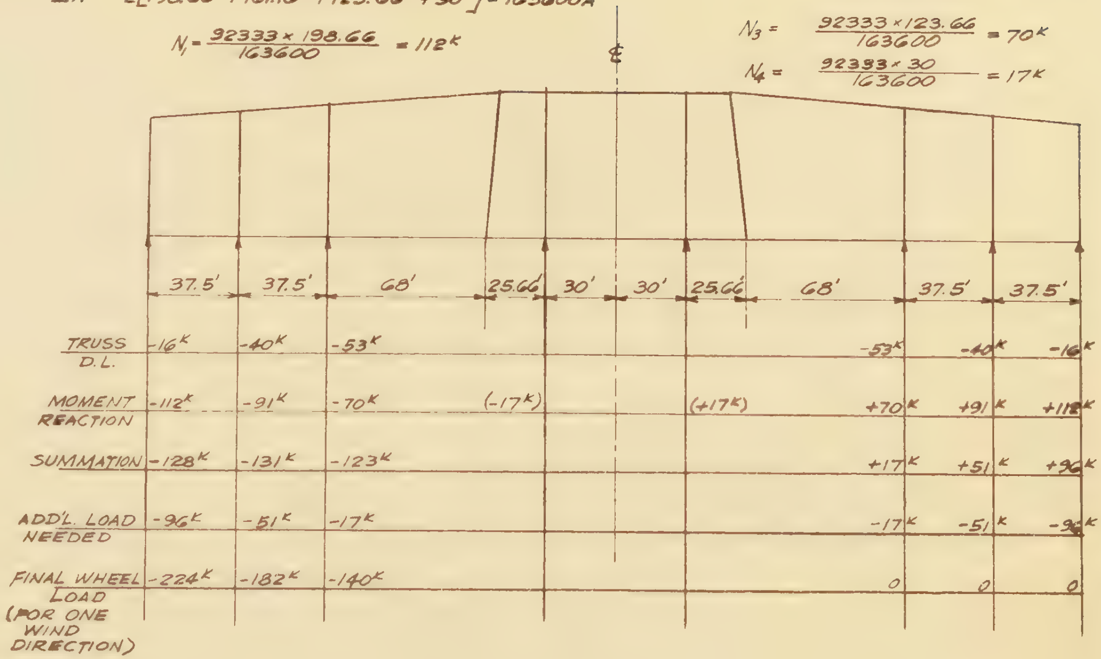
$\Sigma A = 2[198.66^2 + 161.16^2 + 123.66^2 + 30^2] = 163600A$

$N_1 = \dfrac{92333 \times 198.66}{163600} = 112^k$

$N_2 = \dfrac{92333 \times 161.16}{163600} = 91^k$

$N_3 = \dfrac{92333 \times 123.66}{163600} = 70^k$

$N_4 = \dfrac{92333 \times 30}{163600} = 17^k$



TRUSS D.L.	-16^k	-40^k	-53^k			-53^k	-40^k	-16^k
MOMENT REACTION	-112^k	-91^k	-70^k	(-17^k)	$(+17^k)$	$+70^k$	$+91^k$	$+112^k$
SUMMATION	-128^k	-131^k	-123^k			$+17^k$	$+51^k$	$+96^k$
ADD'L. LOAD NEEDED	-96^k	-51^k	-17^k			-17^k	-51^k	-96^k
FINAL WHEEL LOAD (FOR ONE WIND DIRECTION)	-224^k	-182^k	-140^k			0	0	0

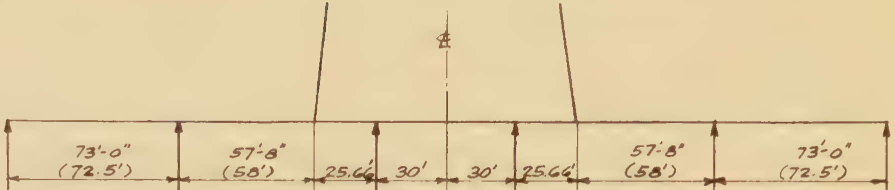
INSIDE TRUSS

$$I.A = 2[186.16^2 + 113.66^2 + 30^2] = 97000A$$

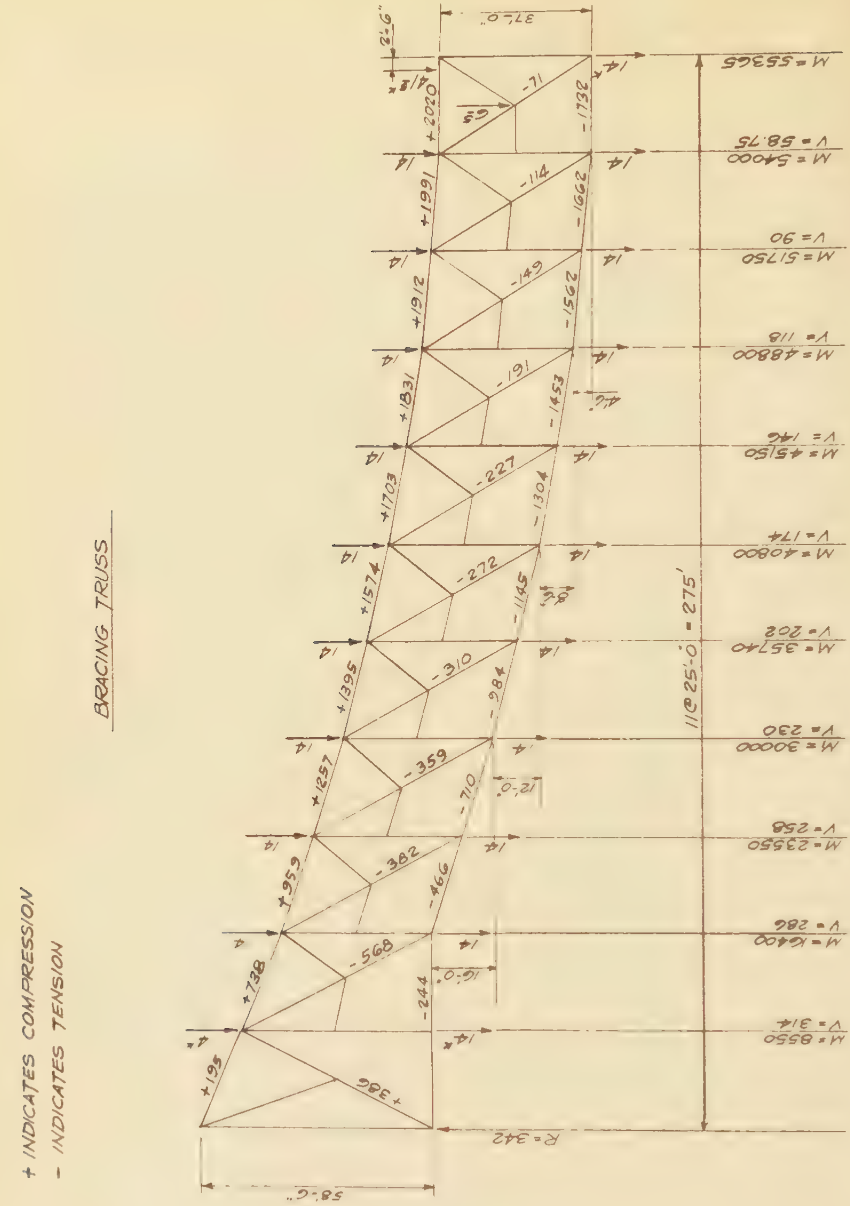
$$N_1 = \frac{92333 \times 186.16}{97000} = 178^k$$

$$N_2 = \frac{92333 \times 113.66}{97000} = 108^k$$

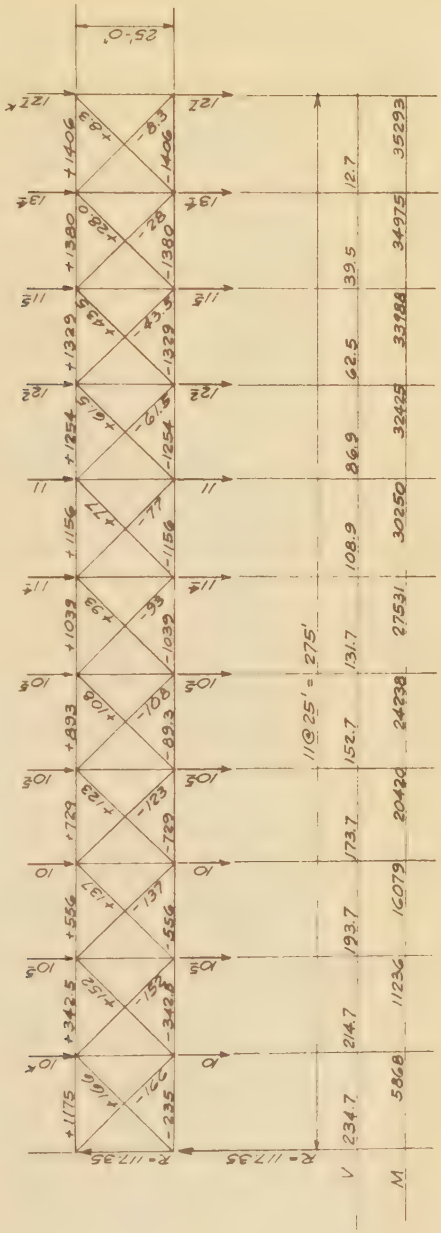
$$N_3 = \frac{92333 \times 30}{97000} = 29^k$$



TRUSS D.L.	-36^k	-65^k			-65^k	-36^k
MOMENT REACTION	-178^k	-108^k	(-29^k)	$(+29^k)$	$+108^k$	$+178^k$
SUMMATION	-214^k	-173^k			$+43^k$	$+142^k$
ADD'L. LOAD NEEDED	-142^k	-43^k			-43^k	-142^k
FINAL WHEEL LOAD (FOR ONE WIND DIRECTION)	-376^k	-216^k			0	0

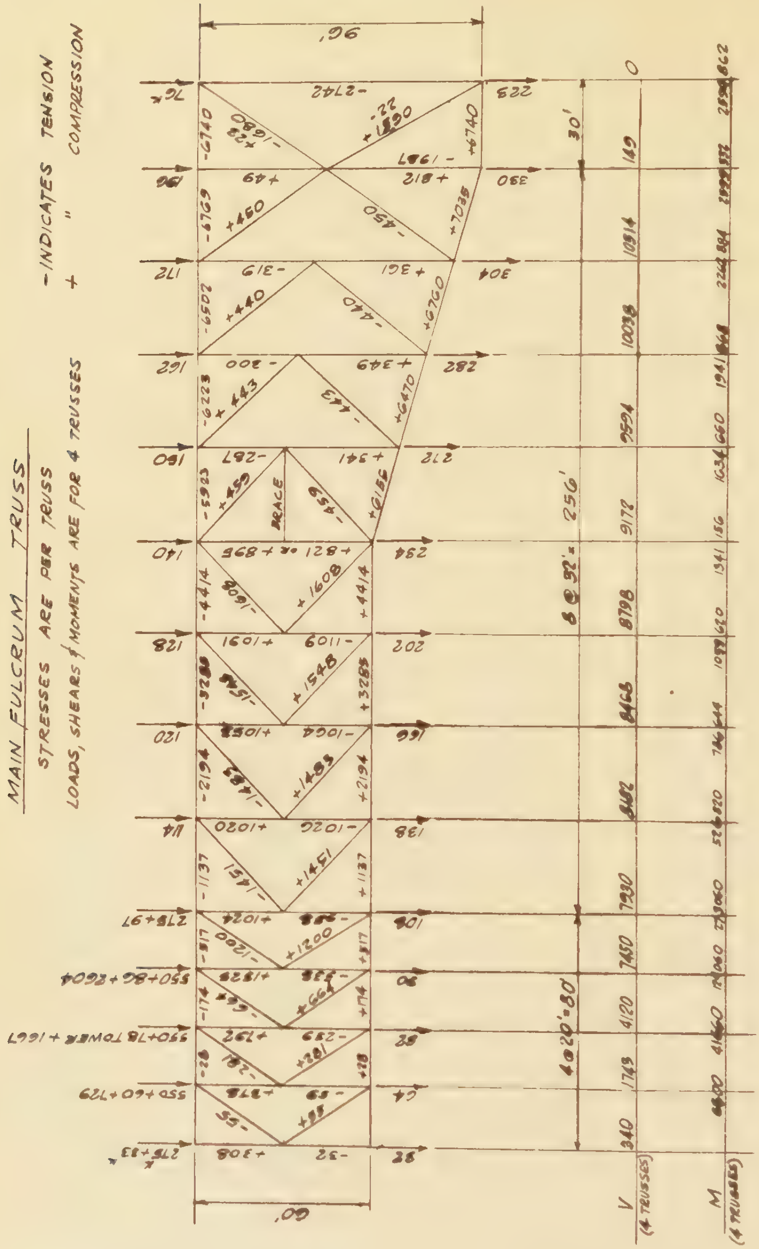


BRACING TRUSS



BRACES IN HORIZONTAL POSITION

+ INDICATES COMPRESSION
- INDICATES TENSION



MAIN FULCRUM TRUSS

STRESSES ARE PER TRUSS

LOADS, SHEARS & MOMENTS ARE FOR 4 TRUSSES

- INDICATES TENSION

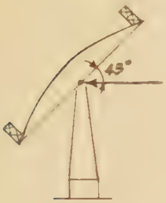
+ " COMPRESSION

MECHANICAL EQUIPMENT

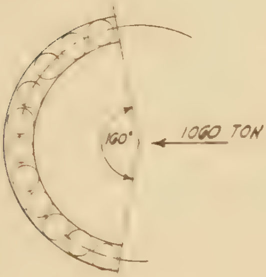
1) CENTER STEDIMENT

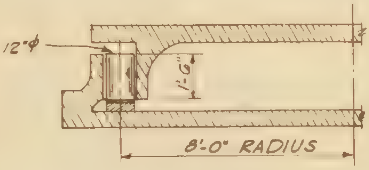
HORIZONTAL REACTION TO BE RESISTED BY CENTER STEDIMENT:

USE WIND LOAD @ 10 PSF



$$H = \frac{0.707 \times 300\,000 \times 10}{2000} = 1060 \text{ TONS}$$





USE: DW × 1000 (DW = DIAMETER × WIDTH)

ROLLER SIZE: $D = 12"\phi$, $W = 1'\text{-}6"$

$12 \times 18 \times 1000 = 216^{k}$/ROLLER = 108 TON/ROLLER [BASED ON FACTOR OF SAFETY = 5]

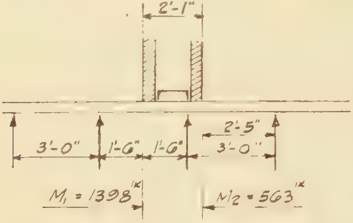
NO. OF ROLLERS FOR 160° SEGMENT $= \frac{1060}{108} =$ 10 ROLLERS

" " " " 360° CIRCLE $= \frac{10 \times 360}{160} = 22.5 \sim 25$ ROLLERS (12"φ × 18")

DESIGN OF RADIAL BEAMS UNDER TRUSS BEARING ON 33"φ ROLLERS AT OUTER TRUSSES. (SEE DWG NO: 9)

CONDITION AT END STIFFENER BEAM:

LOAD = 233^{K}/WHEEL



1) $M_1 = 233 \times 6.0 = 1398^{'K}$

$S = \frac{1398 \times 12}{25} = 671\ IN^3$

⅊ 52" WIDE 3" THICK

S.M. OF ⅊ $= \frac{52 \times \bar{3}^2}{6} = 78\ IN^3$

I OF BM. $= 671 - 78 = 593\ IN^4$

2) SHEAR RESISTANCE OF ⅊

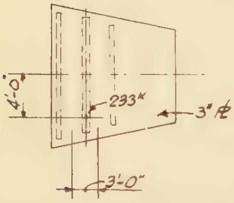
$= 52 \times 3 \times 15 = 2340^{K}$

WEB AREA ON BEAM $= \frac{233}{5} = 15.5\ IN^2$

∴ USE. 36 WF 182

DESIGN OF ROLLER ASSEMBLY AT INNER TRUSSES (SEE DWG NO: 9)

CONDITION AT 2ND BEAM FROM WIDE END OF 3" DISTRIBUTION PLATE:



$M_{MAX} = 233 \times 4.0 = 932^{'K}$

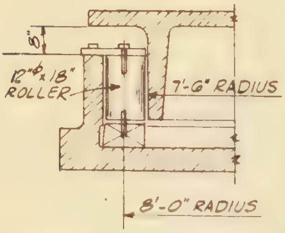
$S = \frac{932 \times 12}{25} = 448\ IN^3$

S.M. OF ⅊ $= \frac{36 \times \bar{3}^2}{6} = 54\ IN^3$ FOR 3" ⅊

S.M. OF BEAM $= 448 - 54 = 394\ IN^3$

∴ USE: 36 WF 150

DESIGN OF CENTER THRUST BEARING (SEE SECTION H-H, DWG NO: 9)



ANGULAR VELOCITY = 35° PER MINUTE

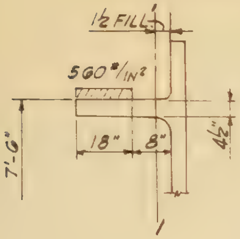
TOTAL MASS = $22\,200^{K}$. CENTRIFUGAL FORCE BASED ON ROTATION IS NEGLIGIBLE. USE 10% OF TOTAL MASS AS THRUST OR 2500^{K} APPROXIMATELY

ROLLERS 12×18 @ $\frac{1}{2}$ TON/FT² = 108 TON/ROLLER

25 ROLLERS $= 108 \times 25 = 5400^{K}$ ON CIRCUMFERENCE OR 3200 ON 160° (11 ROLLERS)

ACTUAL LOAD = 2500/11 = 227^{K}/ROLLER, SPACED 24" O.C. FOR ROLLERS ON 8'-0" RADIUS. (FOR PRESSURE ON INNER SHELL, USE 7'-6" RADIUS AND LOADS AT 22.6

PRESSURE $= \frac{227}{22.6"} = 10.07^{K}$/IN ON 18" HEIGHT OR 560 LB/IN²

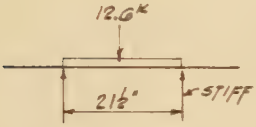


$M_{1-1} = 560 \times 18 \times 15.5 = 156500^{1\#}$

$S = \frac{156500}{20000} = \frac{t^2}{6} = 7.8\,IN^3 \quad t = \sqrt{7.8 \times 6} = 6.83''$

$v = \frac{10.07}{4.5} = 2.24\ ^{K}/IN^2$

BUT, USING 25 RADIAL STIFFENERS SPACED 21½" O.C. ON 7'-1½" RADIUS

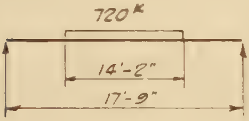


ASSUME CONCENTRATED LOAD OF $\frac{227}{18} = 12.6^{K}$

$M = 12.6 \times 21.5 \times \frac{1}{8} = 33800$ IN. LB.

$t = \sqrt{\frac{6 \times 33800}{20\,000}} = 3.20''$ USE: 4½"

DESIGN OF TRANSFER GIRDERS FOR THRUST

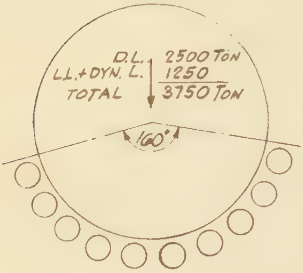


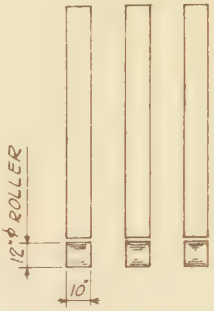
$M = 360 \times 5.34 = 1920^{1K}$

$S = \frac{1920 \times 12}{25} = 922\,IN^3$

USE: 4 ⊔ GIRDERS
WEB 36 × ¾"
FILL PLS 16 × 1¾"
STIFFENERS ⅝" - 5'-0" O.C.

2) THE TRUNNION





USE: DW × 2000

ROLLER SIZE: D = 12"ϕ, W = 10"

$\frac{10 \times 12 \times 2000}{2000} = 120 \frac{\text{TON}}{\text{ROLLER}}$

NO. OF ROLLERS $= \frac{3750}{120}$

$= 31.4$ ROLLERS

ARRANGE 10 @ 3 TIRES

= 30 ROLLERS

∴ 10 @ 3 TIRES AT 160°

USE: 2 BEARINGS AT EA. ROLLER

ROLLING FRICTION

ASSUME 20 #/ TON

D.L.	2500 × 20 =	50000#
L.L.	1250 × 20 =	25000
		75000#

ASSUME SPEED OF 1 REVOLUTION IN 30 MINUTES

TRUNNION DIAMETER = 20 FTϕ   CIRCUMFERENCE = 3.14 × 20 = 62.8 FT

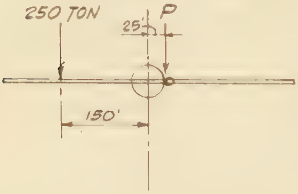
SPEED $= \frac{62.8 \text{ FT}}{30 \text{ MIN}} = 2.09$ FT/MIN

POWER $= \frac{pv}{27000} = \frac{75000 \times 2.09}{27000} = 5.8$ HP   EFFICIENCY $= \frac{27000}{33000} = 0.82$

∴ 5.8 HP REQ'D FOR ROTATION AGAINST ROLLING FRICTION

RESISTANCE TO WIND

ASSUME WIND RESISTANCE AS FOLLOWS:



TORQUE = 250 × 150 = 37500 FOOT-TONS

P = FORCE AT PINION $= \frac{37500}{25} = 1500$ TONS = 3000000 LB

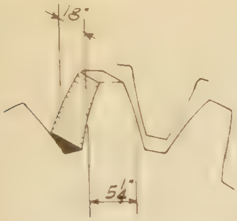
GEAR CIRCUMFERENCE = 50 × 3.14 = 157 FT

30 MIN/REV $v = \frac{157}{30} = 5.23$ FPM

POWER $= \frac{3000000 \times 5.23}{27000} = 581$ HP

USE: 4 MOTORS @ 150 HP

STRENGTH OF TEETH TO RESIST TORQUE IN GEAR



$5\frac{1}{4} \times 18 = 94$ IN^2

4 POINTS OF RESISTANCE (2 ON EA. TOWER)

$94 \times 4 = 376$ IN^2

USE: ALLOWABLE SHEAR VALUE OF
20 KSI = 10 TON/IN^2

ALLOWABLE $P = 376 \times 10 = 3760$ TON > P ACTUAL = 1500 TON
(SEE SHEET 75)

3) AZIMUTH ROTATION

TRUCKS

USE 4 WHEEL TRUCKS AND 6 WHEEL TRUCKS

WHEEL DIMENSIONS: 33" DIA. × 4" WIDTH

RESISTANCE OF ONE WHEEL @ 1000 DW:

$33 \times 4 \times 1000 = 132\,000^{\#} = 132^{k} = 66$ TONS

LOAD CAPACITY OF 4 WHEEL TRUCK = $66 \times 4 = 264$ TONS
" " " 6 " " = $66 \times 6 = 396$ TONS

USE: $\begin{cases} 2 \times 6 & \text{- 4 WHEEL TRUCKS} \\ +4 \times 7 & \text{- 6 WHEEL TRUCKS} \end{cases}$ FOR ARRANGEMENT OF TRUCKS SEE DWG #9

D.L. ON TRUCKS = 2500 TONS

RESISTANCE OF 12 - 4 WHEEL TRUCKS = $12 \times 264 = 3170$ TONS > 2500 TONS

MAX. LOAD ON EXTEROIR TRUCKS DUE TO WIND LOADS
$= 376^{k} = 188$ TONS < 396 TONS (SEE SHEET 68)

TWO ROTATION SPEEDS

SPEED A: $v_A = 50$ FPM (SLOW SPEED)

SPEED B: $v_B = 200$ FPM (HIGH SPEED)

EFFICIENCY = 25000/33000 = 75 %

ROLLING FRICTION @ 20 LB/TON $P = 2500 \text{ TONS} \times 20 = 50\,000$ LB

$$\text{POWER}_A = \frac{P v}{33000 \times \text{EFF.}} = \frac{50000 \times 50}{33000 \times 0.75} = 100 \text{ HP}$$ USE: 4 @ 25 HP

$$\text{POWER}_B = \frac{50000 \times 200}{33000 \times 0.75} = 400 \text{ HP}$$ USE: 8 @ 50 HP

MATERIALS AND EQUIPMENT SELECTED

For the Shell

(1) The skin is $\frac{1}{16}$-in. rolled Anoclad type 20 aluminum sheet* covered on both faces with Alcoa Architectural Gray 2020 finish.* This surface, of some 17 types tested by the Alcoa Laboratories at New Kensington, Pa., showed the lowest total reflectance of white light (14.5 per cent), with a specular reflectance at 0.55 μ at an angle of 45° of 2.40 per cent. The next best sample was a sand-blasted 3003 alloy sheet with 2 coats of methacrylate lacquer that showed 22.5 per cent total reflectance and 2.07 per cent specular reflectance. The Alcoa 2020 finish was used on the Alcoa Building in Pittsburgh, Pa., and on 99 Park Avenue and 460 Park Avenue in New York, N. Y. Several years' testing of this material for sunfastness shows very little noticeable change in color due to exposure. The emissivity of this sheet is about 80 per cent. With this surface there will be little concentration of photon energy at the focal point, even when the shell is pointed directly into the sun and there is almost instantaneous balancing of temperatures on the 2 faces of the shell.

(2) The steel gratings are of standard model and dimension and will be zinc plated electrically at the factory during fabrication. Surfaces disturbed by the field welding will be recoated by metalizing.

(3) The steel-grid framework, ring, and cross girder will be of structural steel A-7 A.S.T.M.;† the use of the higher-strength steels is not warranted since they have the same elastic modulus as this type, and since stresses are extremely low in the make-up of the shell, which is controlled by the rigidity required. All steel faces will be metalized with zinc; the major areas will be so treated before erection, and the areas that are disturbed by welding will be repaired after assembly.

(4) All joints in the framing will be welded to reduce corrosion at the exposed edges to eliminate the added weight of connections and to reduce wind resistance.

(5) Steel surfaces will be grit blasted with angular steel grit of S.A.E.‡ G-25 mesh size under 75 psi air pressure. Metalizing can be with aluminum alloy 4043, providing 0.007-in. covering, or with zinc, providing 0.004 to 0.006 in. covering. The latter is recommended by the metalizing industry, and is estimated to cost not over $0.25 per square foot of surface (about $30.00 per ton of structural steel). This treatment will eliminate all painting and should serve without maintenance for at least 25 and, possibly, 50 years. It is suggested that all other structural steel in the installation be so protected.

(6) Cables and threaded cable-adjustable ends are to be Roebling high-strength prestressed cables made for use in prestressed concrete with a guaranteed elastic modulus of 24,000,000 psi and a working strength of over 200,000 psi.

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* Products of the Aluminum Company of America, New York, N. Y.

† American Society for Testing Materials, Philadelphia, Pa.

‡ Society of Automotive Engineers, New York, N. Y.

For the Towers and Support Trusses

(1) The structural steel prescribed is high-strength alloy steel, A 242 A.S.T.M. with 50,000 psi minimum yield point to reduce the dead weights needed to allow for higher permissible working stresses.

(2) Cables same as item 6 above.

(3) Concrete in counterweights to be densest mix available locally.

For the Focal-Point Mast

(1) Mainmast to be structural-steel seamless pipe; (2) the upper section o be glass-fiber tube; and (3) the guys to be high-strength prestressed steel cables.

For the Trunnions

(1) The main shaft is to be of high-strength alloy-steel billet welded and finished to surface.

(2) The rollers are to be finished cylinders of roller-bearing steel, each on 2 bearings to provide a rolling-friction resistance of 20 lb. per ton of load and a support of 120 tons per roller

(3) Gears and pinions are to be welded assemblies of flame-cut sections. Units of this type were made by Derrick and Hoist, Inc., Long Island City, N. Y., for the United States Navy for rotation control of a gantry crane. Cutting through a 6-in. thickness of 1040 rolled-steel plate is controlled by an Airco Oxygraph and Electronic Tracer, and progress was 7 in. per min. The accuracy of the cut exceeded the tolerance permitted for cast-steel gears.[6]

(4) The housing and base are welded steel plate assemblies.

(5) The gear reducers are Jones* complete units.

(6) The motors are 1800 rpm, 440 V, equipped with radio interference reducers as per Joint Army and Navy capacitor specification. Interference not to exceed 10.0 μ volts per meter over the band from 60 kilocycles per second to 400 megacycles per second.

(7) Hydraulic and Eddy current brakes are supplied on all motor shafts.

(8) Gear reduction provided is 1:54,000, so that normal speed of rotation of shell is about 12° per minute. For tracking speed, an intermediate vernier or gear reducer of 1:50 will be required, equivalent to 0.109 ft./min. along the pitch of the bull gear.

For the Azimuth Motion

(1) Standard first-class passenger railroad track as per American Railway Engineering Association (A.R.E.A.) standards as modified for subway roadbeds with a similar concrete base and imbedded ties for the ring cog.

(2) Standard 6-wheel Pullman car trucks for the traction units and 4-wheel railroad car trucks for the idler units.

(3) A ring cog gear on 652-foot circle, with vertical faces 12 inches high,

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* Product of the W. A. Jones Foundry and Machine Company, Chicago, Ill.

to allow for vertical position variations of the pinions, which will be flam cut from solid metals and assembled by welding.*

(4) Pinions on vertical shafts operated from truck-mounted slow-spee propelling gears and 50-H.P. motors. There are four identical drives o each wind truss carrying a tower. Each unit is equipped with hydraulic an DC brakes.

(5) The motors are standard 1800 rpm, 440 V.†

(6) Gear reduction is provided with a 14½-in. pinion running against th cog gear, and it provides a steady movement of 52.7 ft./min. (9°), or 40 min for a complete rotation. Tracking speed will require a vernier or additiona gear reduction of 1:37 to provide movement along the cog of 1.42 ft./min.

For the Central "Stediment" Support

(1) The concrete foundation should be on rock or such a layer of soil tha a varying load of from 4 tons/sq. ft. to zero will not result in a permanent se or an elastic deformation of 0.01 ft.

(2) For trackage there are four concentric lines of No. 175 crane rail 4 in. wide, with webs reinforced by welded chock ribs.

(3) The rollers are high-strength hardened steel 33 in. in diameter wit beveled flanges to reduce wear against the heads of the rails. They run o 8-in. roller-bearing pins fixed to yokes on which the cross-truss grillage rest and to which it is connected rigidly. Each roller must support 120 tons.

(4) Interlocked top- and bottom-center assemblies with 20 12-i round × 18-in. high vertical rollers and 10 smaller supporting rollers t transmit all lateral forces from the cross truss to the foundation. This co struction is designed to withstand a 100-mph wind acting on the maximu possible exposed surfaces.

Miscellaneous Items

(1) The gear and retraction mechanism to be at the focal point.

(2) The elevators in each tower to be 200 ft./min. push-button automat cars, each having a capacity of 2000 lb.

(3) The hydraulic-plunger lift to be at center of cross truss for access the instrument at the focal point when the shell is inverted.

(4) Zinc-metalized steel stairs are to be constructed in each tower and the center of the cross truss, with safety-type grating walkways connecti all stairs.

(5) An enclosed platform to be provided at each trunnion-control area the tops of the towers.

(6) The observation room to be under the center of the shell. There w be protected walkway connection passageways from each tower to this roo

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* See item 3 under "*For the Trunnions.*"

† See item 6 under "*For the Trunnions.*"

COST ESTIMATE TABLES (*as of* 1955)

TABLE 1
ESTIMATE OF QUANTITIES AND COSTS

		Material	*Labor*		
Shell:					
$^{1}{}_{25}$-in. aluminum skin (85 tons)	300,000 sq. ft. @	$0.60	$0.10	$210,000	
Steel gratings (710 tons)	544,000 sq. ft. @	1.35	0.25	870,000	
Steel joist grid	740 tons @	160	200	266,400	
Structural work	2,614 tons @	180	200	990,000	
Metalizing	3,350 tons @		30	100,000	
Cables and connectors (14,500 linear ft.)	87,000 lb. @	1.00/ft.	0.20/lb.	32,000	
Lifting and positioning	4,150 tons @		40–	150,000	
					$2,623,000
. Towers and girders:					
Structural work	4,762 tons @	200	200	1,905,000	
Metalizing	4,762 tons		20	96,000	
Cables (72,250 linear ft.)	217 tons @	320	200	113,000	
					$2,114,000
. Foundation work:					
Excavation (estimated)	250,000 cu. yd. @		2	500,000	
Concrete work				1,125,000	
Rails	214 tons of 100 lb. @	100		27,000	
	45 tons of 175 lb. @	120			
Trackage	6,000 linear ft. @	10	10	120,000	
					$1,772,000
. Equipment: (see TABLE 3)					1,900,000
Total					8,409,000
Contingency on costs 7%					591,000
Total estimated cost					$9,000,000

otes:

(1)	Total structural weight	9,214 tons @ 600	$5,525,000
	Total mechanical weight	700 tons @ 2,000	1,400,000
	Electrical and control items		400,000
	Excavation and concrete		1,745,000
			$9,070,000

(2) Concrete work includes:
Track base 63 ft. wide × 3 ft. thick: 15,400 cu. yd.
Center footing 90 × 90 × 12 ft.: 3,600 cu. yd.
Counterweights (4 @ 1,500): 6,000 cu. yd.

(3) Aluminum sheet costs:
Basic material 0.040 in. thick: 0.27/sq. ft.
2 finish surfaces: 0.33
0.66/sq. ft.

(4) No allowance for:
(a) Electrical controls for automatic operations
(b) Power facilities
(c) Housing and offices
(d) Access, water, drainage, fire protection, and communications

TABLE 2
COST ESTIMATE FOR MATERIALS OF SUPERSTRUCTURE

Shell:	
$\frac{1}{25}$ in. ± aluminum skin	
302,000 sq. ft. × 0.56 psf = 175,000 lb. =	85.0 tons
Zinc-coated steel grating	
1 × $1\frac{1}{8}$ in. bracing bars @ $4\frac{3}{4}$ in. on center	
4 in. ϕ T and B @ 4 in. on center	
Top 600 feet ϕ = 302,000 sq. ft.	
Bottom 550 feet ϕ = 242,000	
544,000 sq. ft. × 2.34 psf = 1,270,000 lb. =	635 tons
Steel joists SJ166	
302,000 sq. ft. × 4.9 psf = 1,480,000 lb. =	740
	1,375 tons
Ring truss: 3760^K =	1,880 tons
Bracing trusses: 920^K =	460
Adjustment cable fittings: 7.4^K =	4
Ring assembly: 4.73^K =	3
Obs. room fl. & clg: 18.3^K =	9
Passageway: 83.6^K =	42
Mast and support: 39.1^K =	20
2 shafts 18 ft 3 in. O.D., 3 in. thick: 345^K =	173
2 shafts 34 ft. 0 in. O.D., 1 in. thick: 46^K =	23
	2,614 tons
Towers, fulcrum, and wing trusses:	
2 towers:	2,160 tons
2 wing assemblies:	220
Main fulcrum trusses:	2,010
Main fulcrum saddles:	98
Main fulcrum anchors:	17
Roller and yoke assembly at trusses at center stediment:	132
Thrust bearing and rotor at center: 249^K =	125
	4,762 tons
Rails:	
Exterior: 12,840 ft. of A.R.A. 100-lb. rail =	214 tons
Interior: 1,546 ft. of A.R.A. 175-lb. rail =	45
	259 tons
Cables:	
Cantilever and brace: 72,240 ft.	
Adjustment: 10,450	
Steel guy: 3,920	
86,610 ft. × 6-lb./ft. = 520,000-lb. =	260 tons

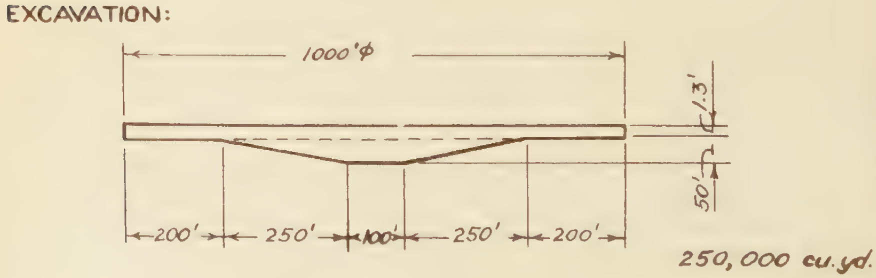

FIGURE 13.

TABLE 3

COST ESTIMATE FOR EQUIPMENT

enter support:	
96 33-in. × 4-in. wheels @ $250:	$ 24,000
96 bearings, bushings, and yokes @ $300:	28,800
96 8-in. pins @ $200:	19,200
20 rollers—12 in. round × 18 in. long @ $450:	9,000
12 roller supports and cage @ $25:	300
Top and bottom assemblies machined—40 tons @ $1,000 per ton:	40,000
	$ 121,300
runnions:	
2 × 72 rollers; 12 in. × 12 in.—28 tons @ $1,600 per ton:	$ 45,000
2 × 72 SKF No. 22324 bearings @ $225:	32,000
2 × 3 rotating bands—5,000 lb. each @ $0.80 per lb.:	24,000
2 bases, welded and annealed—40 tons @ $600 per ton:	24,000
2 trunnion castings—400 tons at $1,000 per ton:	400,000
2 hold-downs	10,000
	$ 535,000
otation mechanism:	
2 50-foot ring gears—180 tons @ $1,000 per ton:	$ 180,000
2 24-in. round × 15-in. wide pinions—6,000 lb. @ $0.75 per lb.:	18,000
8 16-in. round bearings @ $2,000:	16,000
4 gears—6,000 lb. @ $0.625 per lb.:	15,000
4 pinions—18 in. round × 12 in. wide @ $2,500:	10,000
4 gear reducers—585 triple herringbone @ $18,000:	72,000
4 couplings and shafts @ $2,000:	8,000
4 150-H.P., 440-V, 1800-rpm motors @ $3,000:	12,000
4 Eddy current brakes @ $2,500:	10,000
4 150-ton hydraulic brakes @ $1,500:	6,000
	$ 347,000
zimuth mechanism:	
12 standard 4-wheel railroad trucks @ $3,500:	$ 42,000
28 6-wheel standard Pullman trucks @ $5,000:	140,000
2 × 4 slow-speed propelling gears @ $4,000	32,000
2 × 4 pinions and shafts @ $3,000:	24,000
1 ring cog gear 652 feet ϕ—125 tons @ $800 per ton:	100,000
8 50-H.P., 1,800-rpm, 440-V motors @ $1,000:	8,000
8 DC brakes and hydraulic brakes @ $1,500:	12,000
8 couplings @ $250:	2,000
	$ 360,000
iscellaneous:	
1 focal-point adjustment:	$ 10,000
1 Servo cable adjuster for shell:	100,000
2 elevators and 1 hydraulic lift:	50,000
Stairs and walkways—2,000 feet @ $30 per foot:	60,000
	$ 220,000
Total equipment:	$1,583,800
Installation and electrical work—25 per cent:	316,200
Total:	$1,900,000

References

1. Dischinger, F. 1935. Die Rotationsschalen mit unsymmetrisher Form und Belastung. Bauingenieur. Vol. 35/36, 37/38.
2. Dischinger, F. 1928. Schalen und Rippenkuppeln. Handbuch fur Eisenbetonbau Band VI. Wilhelm Ernst & Son. Berlin, Germany.
3. Timoshenko, S. 1940. Theory of Plates and Shells. McGraw-Hill. New York, N. Y.
4. Timoshenko, S. 1936. Theory of Elastic Stability. McGraw-Hill. New York, N. Y.
5. Sorensen, K. E. 1953. Graphical solution of hydraulic problems. Proc. Am. Soc. Civil Engrs. Vol. 118. Paper 2536.
6. Airco News. 1954. **14**(3): 10. Air Reduction Co., Inc., New York, N. Y.

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